Peukert's Law for Supercapacitors: Physics and Application

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Outline

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- Applicability of Peukert’s Law to Supercapacitors
  - Current Mode
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- Dependence of Supercapacitor Peukert Constant on Voltage, Aging, and Temperature

Peukert's Law for Lead-Acid Batteries

- Peukert's law relates delivered charge to discharge current [pkt]:

$$I^k t = Q_0$$

  - Q_0: nominal capacity rated at a particular discharge current
  - I: actual discharge current
  - t: actual discharge time
  - k: Peukert constant (k>1)
- Delivered charge depends on discharge current: the larger the discharge current, the less the delivered charge.

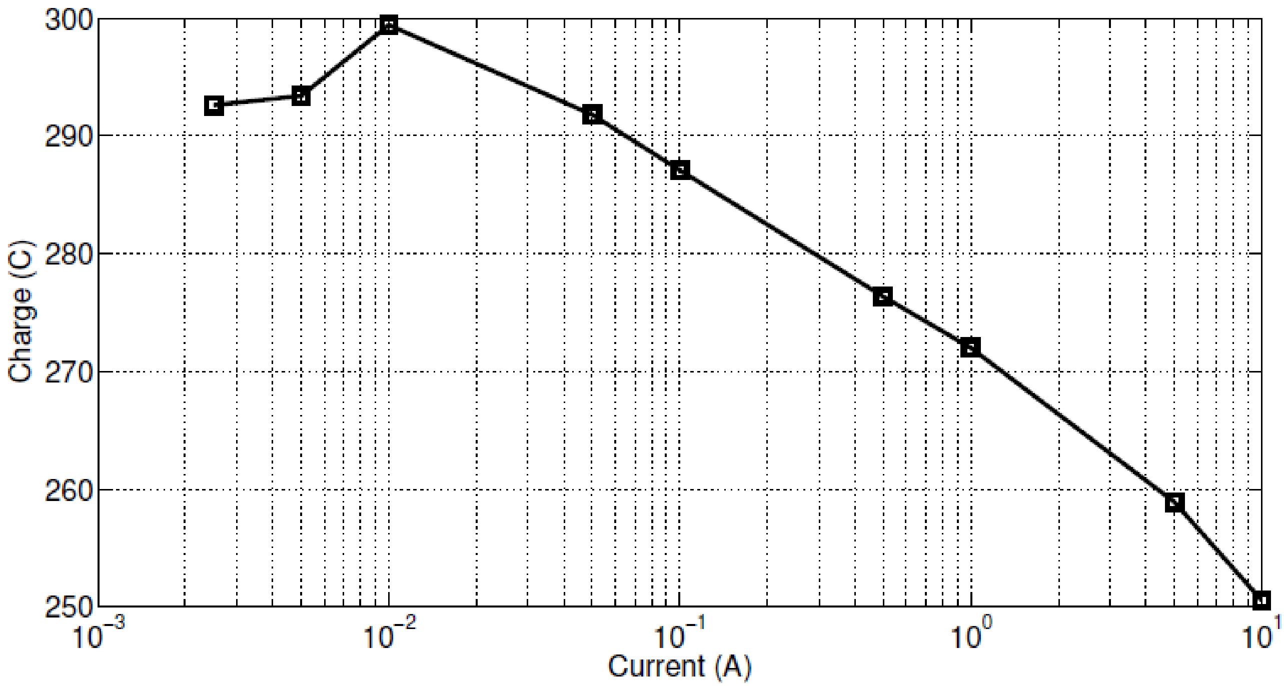
Peukert's Law for Supercapacitors

- Charge delivery capability
  - Relationship between delivered charge and discharge current during a constant current discharge process
- Energy delivery capability
  - Relationship between delivered energy and discharge power during a constant power discharge process
- Peukert's law in current/power modes
  - Applicability: current/power ranges [tpel_cap, tpel_phy, tia]
  - Application in supercapacitor discharge time prediction [jes]
  - Dependence of Peukert constant on terminal voltage, aging condition, and operating temperature [tpel_pkt, jes_pkt]

Applicability: Current Mode

- Delivered charge vs. discharge current (2.7 V/100 F supercapacitor sample, initial voltage 2.7 V, cutoff voltage 0.01 V)
  - 10-0.01 A: Peukert's law applies
  - 0.01-0.0025 A: Peukert's law does not apply
- Peukert's law
  - Q_0: nominal charge
  - I: discharge current
  - k: Peukert constant

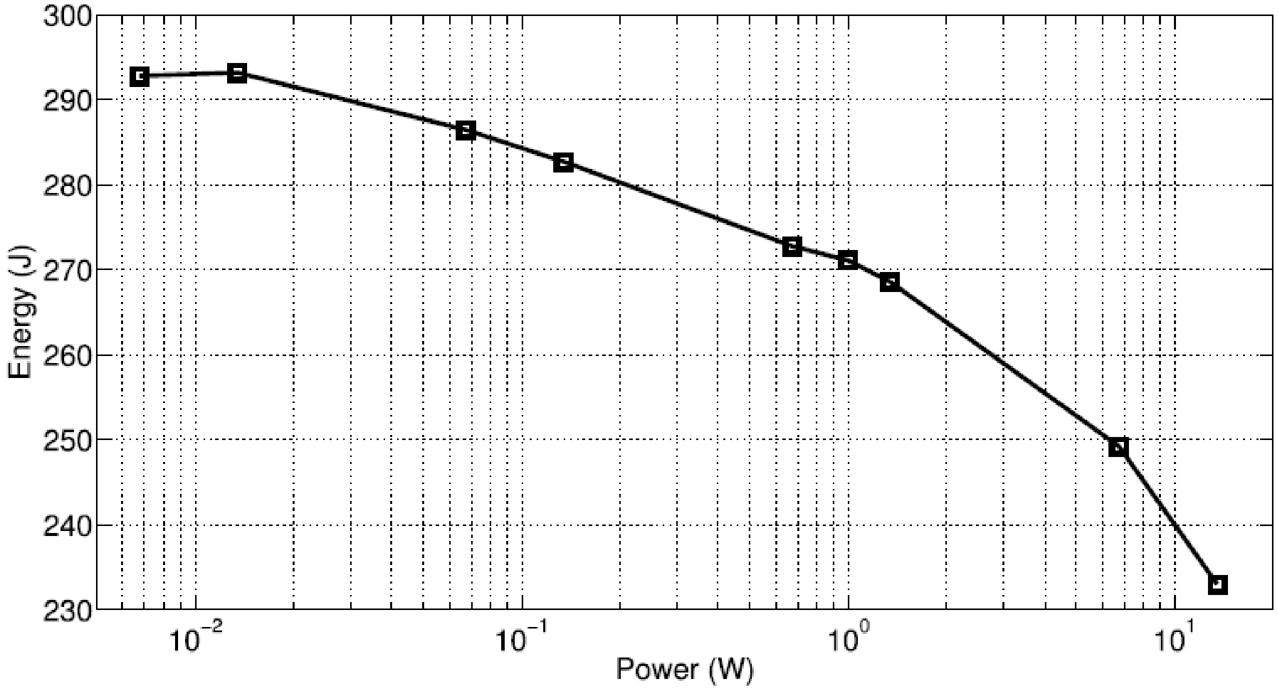
$$I^k t = Q_0$$



Applicability: Power Mode

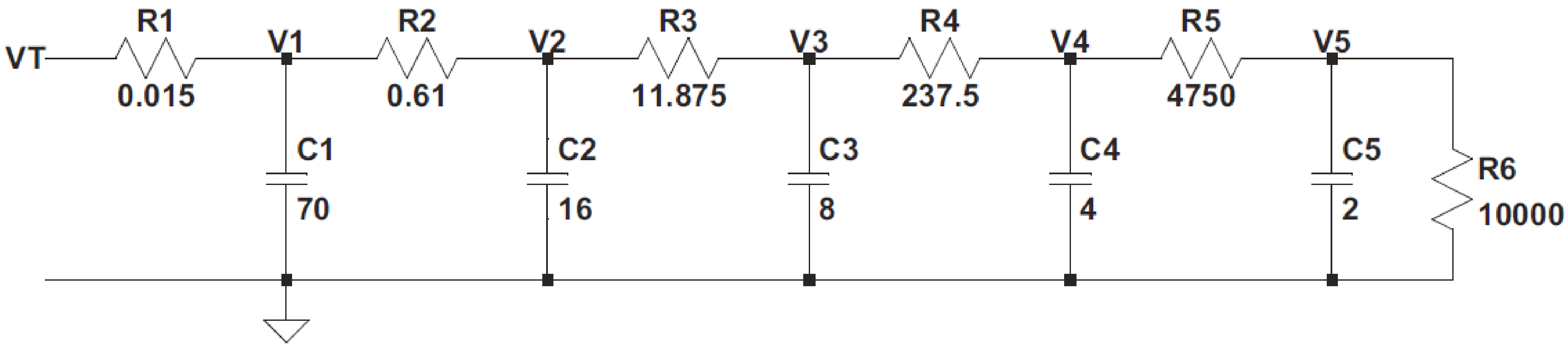
- Delivered energy vs. discharge power (2.7 V/100 F supercapacitor sample, initial voltage 2.7 V, cutoff voltage 1.35 V)
  - 13.5-0.0135 W: Peukert's law applies
  - 0.0135-0.00675 W: Peukert's law does not apply
- Peukert's law
  - E_0: nominal energy
  - P: discharge power
  - k: Peukert constant

$$P^k t = E_0$$



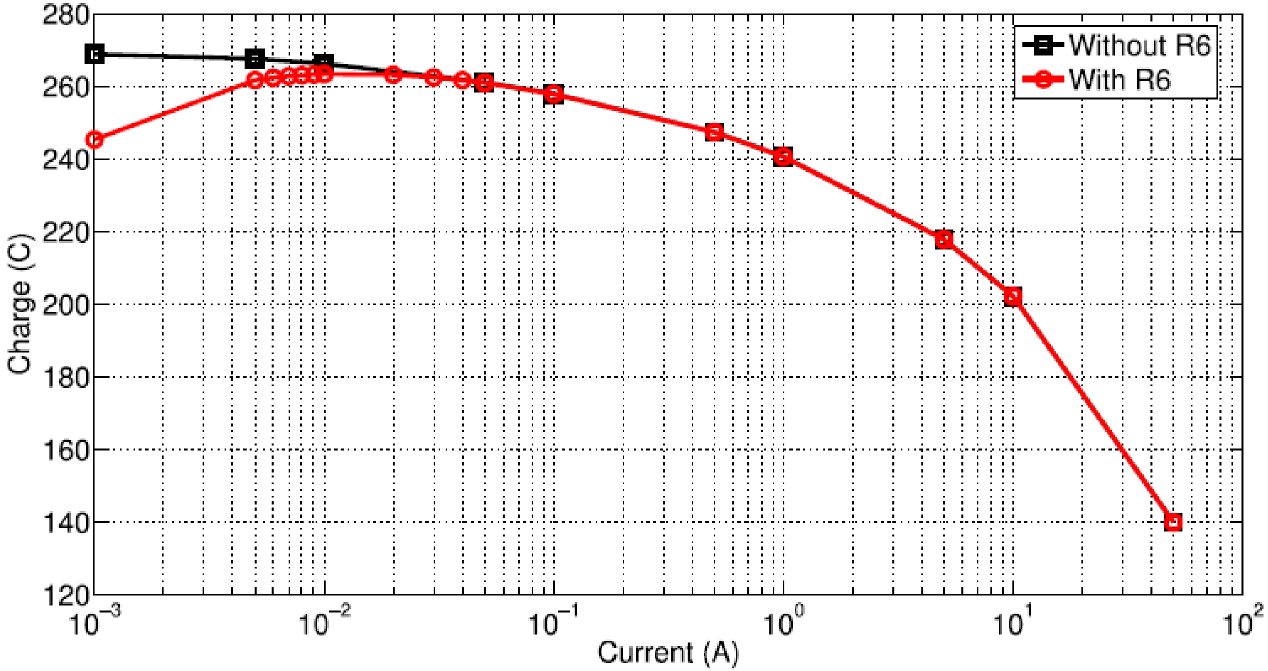
Physical Mechanisms: Supercapacitor Model

- Five-branch *RC* ladder circuit model (2.7 V/100 F supercapacitor)
- Three aspects of supercapacitor physics
  - Porous electrode structure: increase in delivered charge when discharge current decreases (time constants: 1.05, 10, 100, 1000, and 10000 s)
  - Charge redistribution: increase (charge transfer is unidirectional: from slow branches to fast branches)
  - Self-discharge: negligible for large currents (short term), decrease for small currents (long term)



Physical Mechanisms: Simulation Results

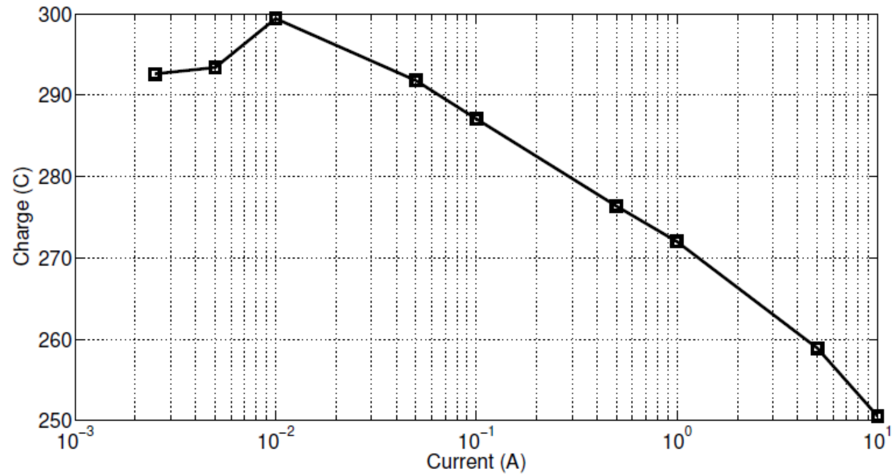
- Delivered charge vs. discharge current (2.7 V/100 F model, initial voltage 2.7 V, cutoff voltage 0.01 V)
  - 50-0.01 A: Peukert's law applies
  - 0.01-0.001 A: Peukert's law does not apply

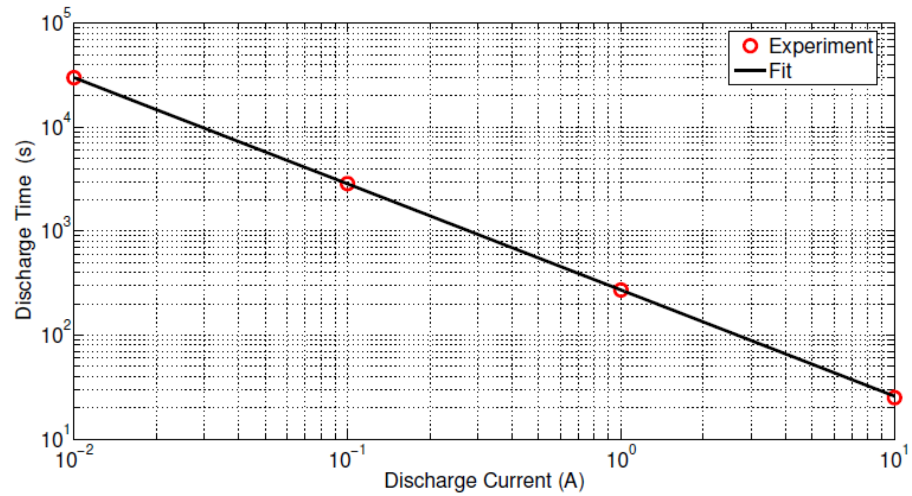


Application Scenario 1: Setup

- Scenario 1: same discharge voltages for training and testing sets
- Setup to fit Peukert constant (applicable 10-0.01 A at 2.7 V)
  - Training set: 10, 1, 0.1, and 0.01 A
  - Testing set: 5, 0.5, and 0.05 A
- Fit function
  - Q_0: charge delivered at 1 A

$$t = \frac{Q_0}{I^k}$$



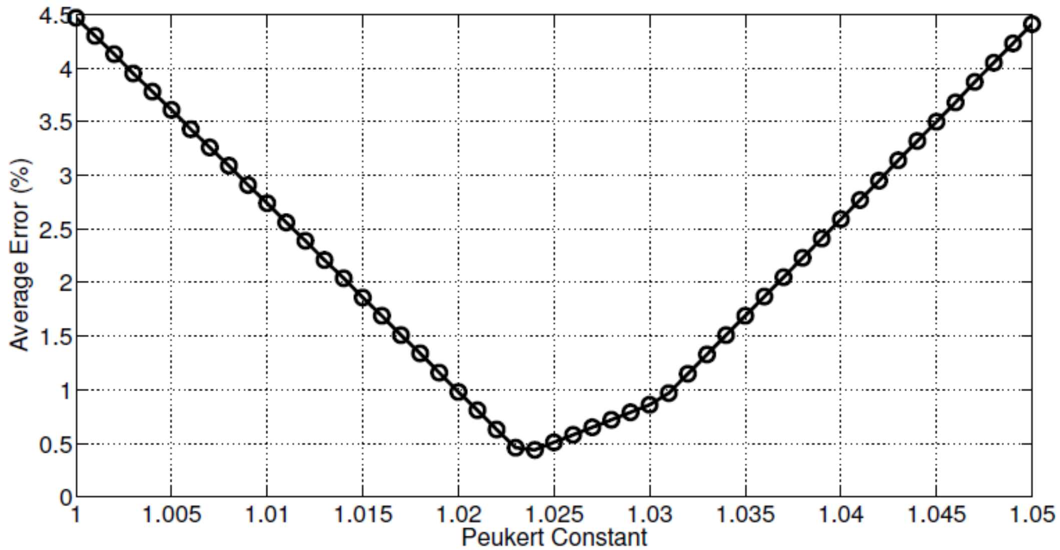


Application Scenario 1: Results

- Prediction error

$$\delta = \frac{|t_p - t_m|}{t_m} \times 100\%$$

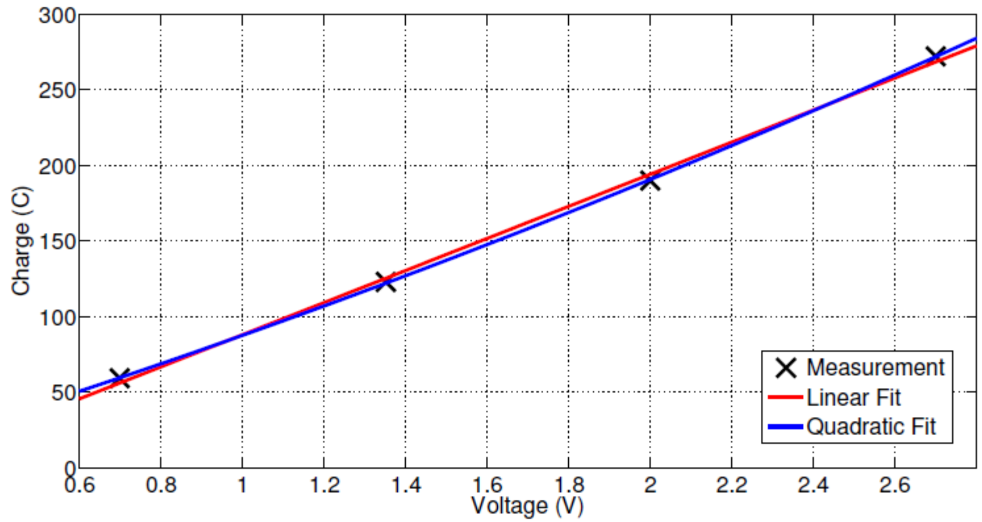
- Rated setup
  - Rated capacitance
- Constant setup
  - Measured Q_0
- Reduced error
  - Fit: ~20%
  - Optimal: ~10%

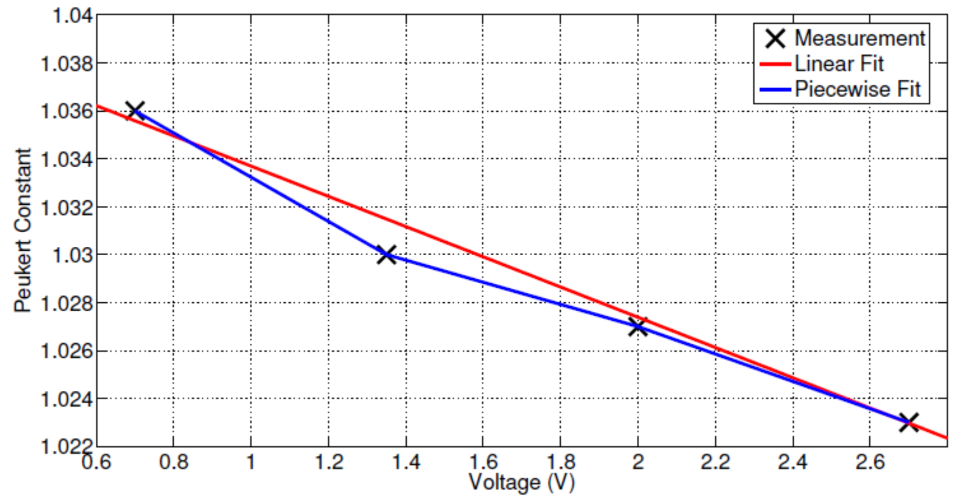


	Rated	Constant	Fit	Optimal
k	1	1	1.021	1.024
Q_0 (C)	269	272.01	272.01	272.01
δ_5 (%)	3.89	5.05	1.56	1.07
$\delta_{0.5}$ (%)	2.67	1.58	0.13	0.07
$\delta_{0.05}$ (%)	7.81	6.78	0.73	0.17
$\overline{\delta}$ (%)	4.79	4.47	0.81	0.44

Application Scenario 2: Setup

- Scenario 2: different discharge voltages for training and testing sets
- Setup to fit nominal charge and Peukert constant
  - Training set: 2.7, 2, 1.35, and 0.7 V
  - Testing set: 2.4, 1.7, and 1 V
- Fit function
  - Q_0: linear and quadratic
  - k: linear and piecewise linear





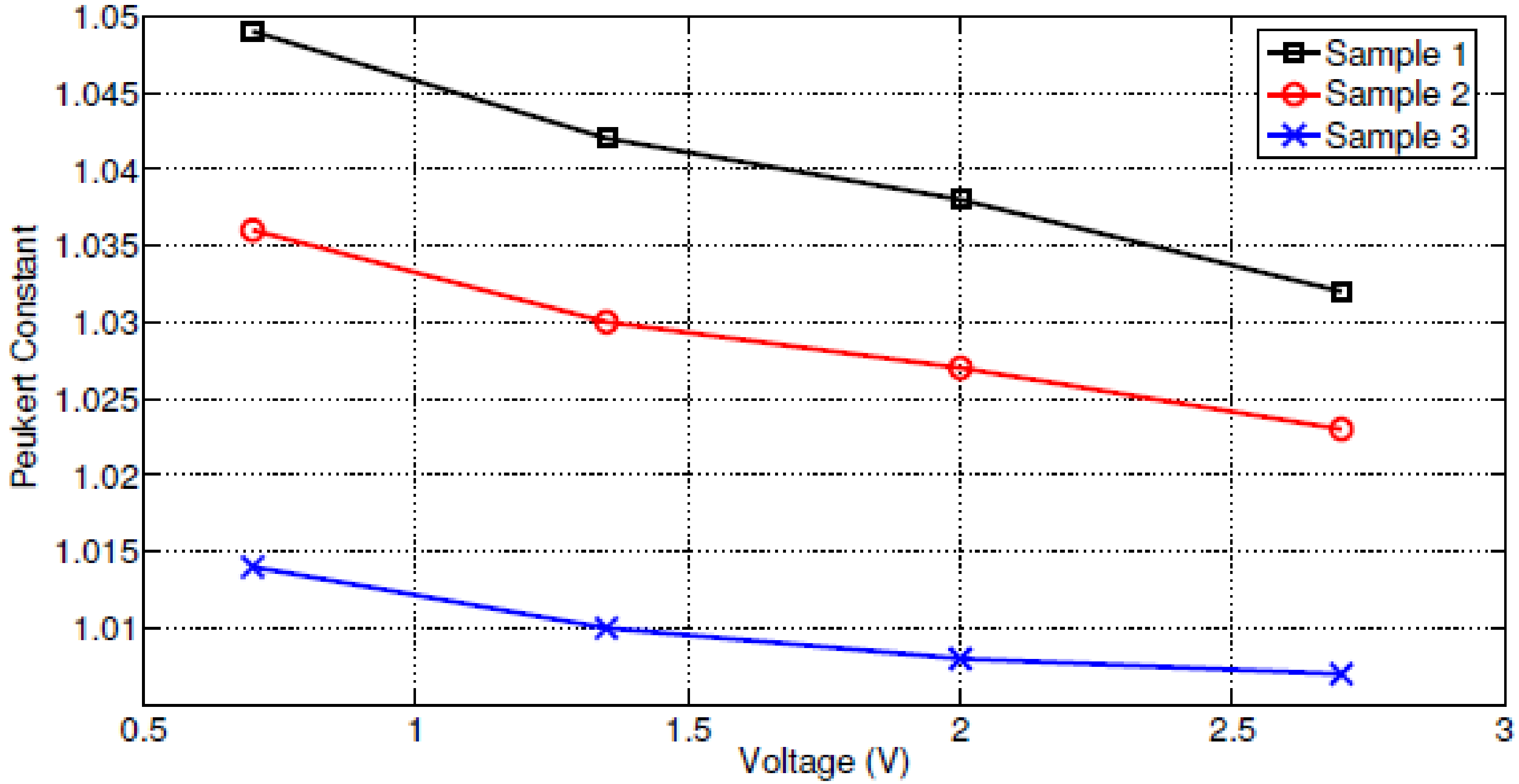
Application Scenario 2: Results

- Rated setup (δ_R)
  - Rated capacitance
- Constant setup (δ_C)
  - Measured/fitted Q_0
- Reduced error
  - Same nominal charge: error is approximately equal
  - Same Peukert constant: error varies significantly with nominal charge

Q_0	k	$\overline{\delta}_R$ (%)	Q_0	k	$\overline{\delta}_C$ (%)	Q_0	k	$\overline{\delta}_F$ (%)	$\overline{\delta}_O$ (%)
R	1	10.20	M	1	10.65	M	M	2.75	2.66
						M	L	2.82	-
						M	P	2.69	-
			L	1	10.27	L	M	4.33	4.50
						L	L	4.75	-
						L	P	4.62	-
			Q	1	10.52	Q	M	3.45	3.42
						Q	L	3.60	-
						Q	P	3.49	-

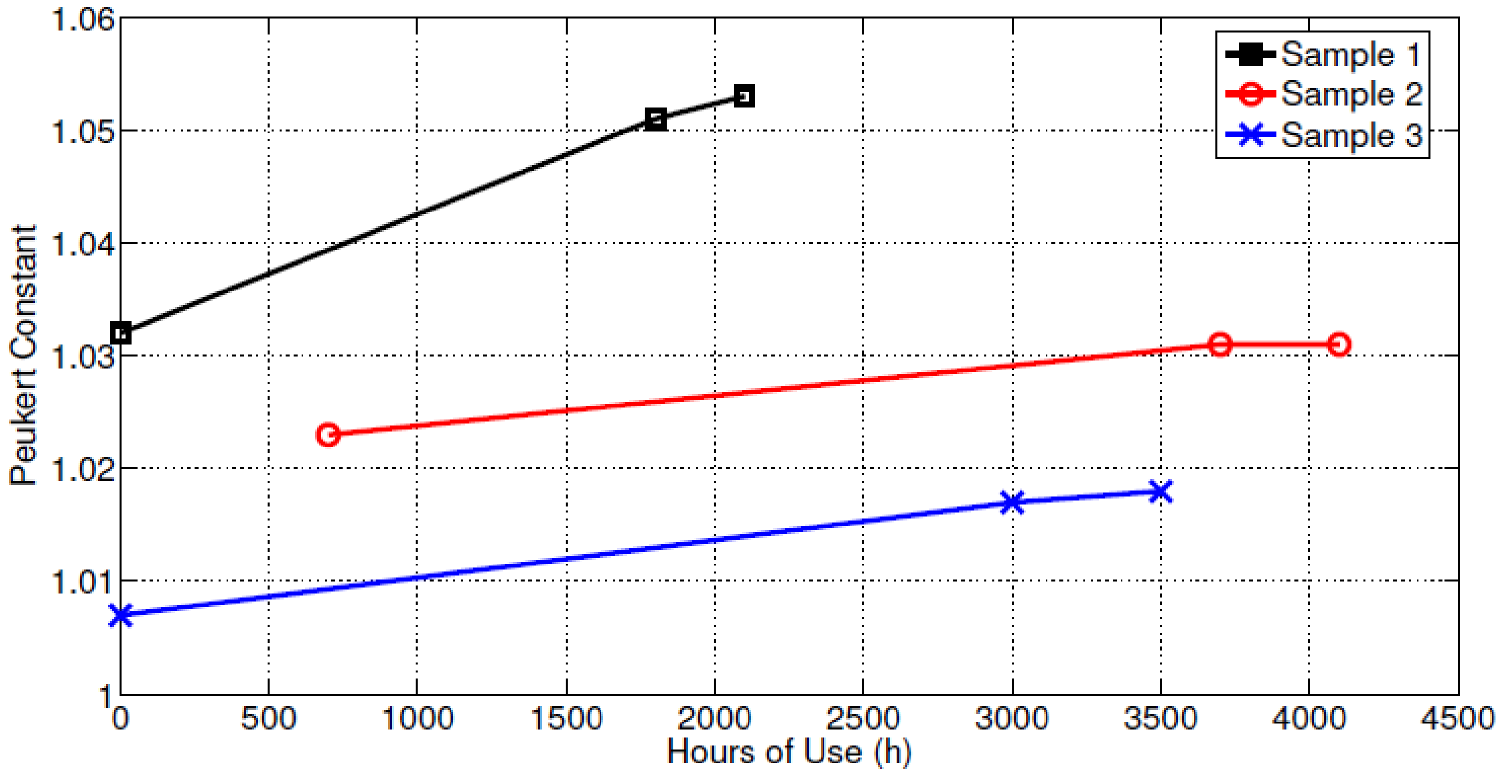
Peukert Constant Dependence on Voltage

- Peukert constant decreases when initial voltage of discharge process increases (cutoff voltage is 0.01 V)



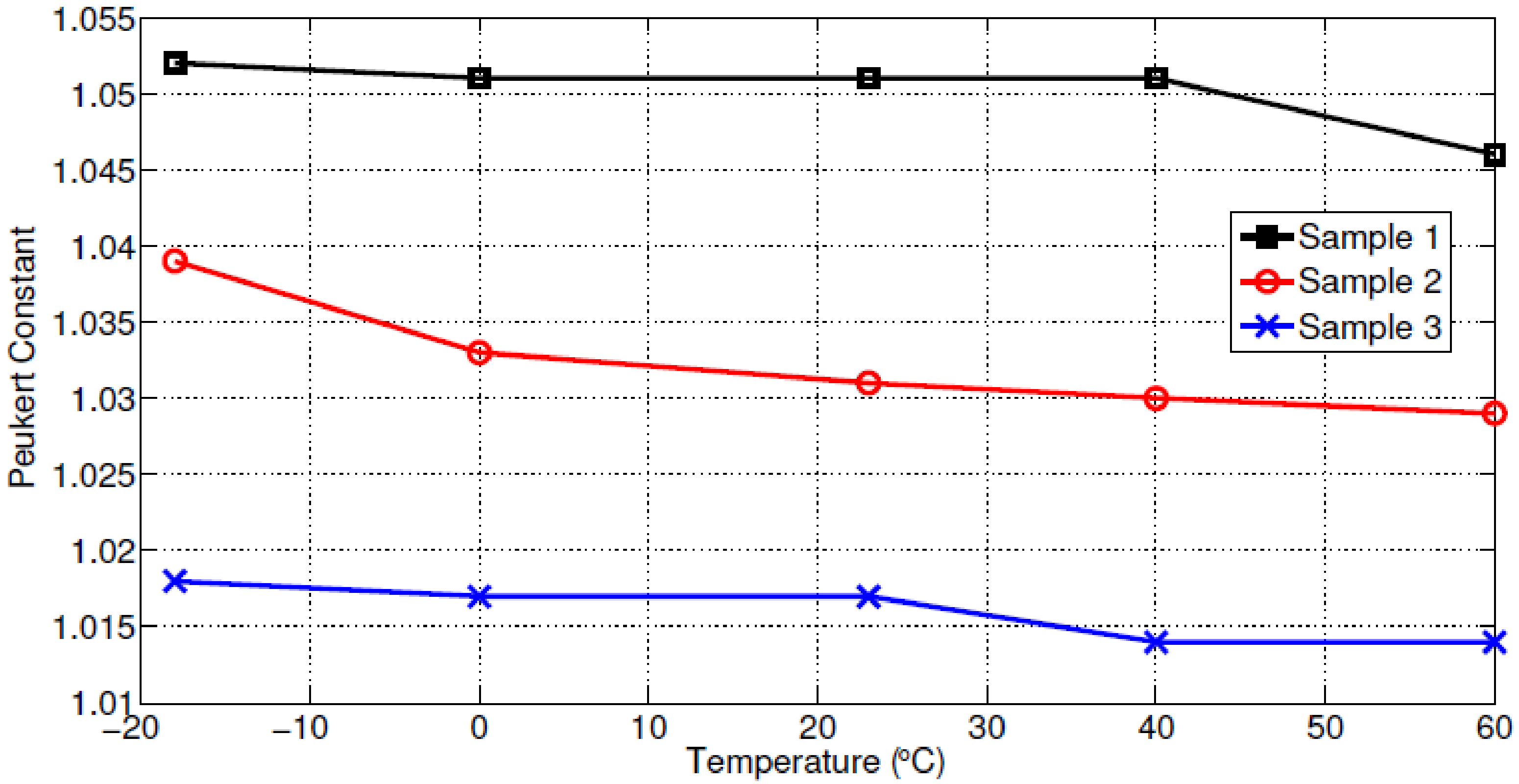
Peukert Constant Dependence on Aging

- Peukert constant increases when supercapacitor is more heavily aged (initial voltage is 2.7 V and cutoff voltage is 0.01 V)



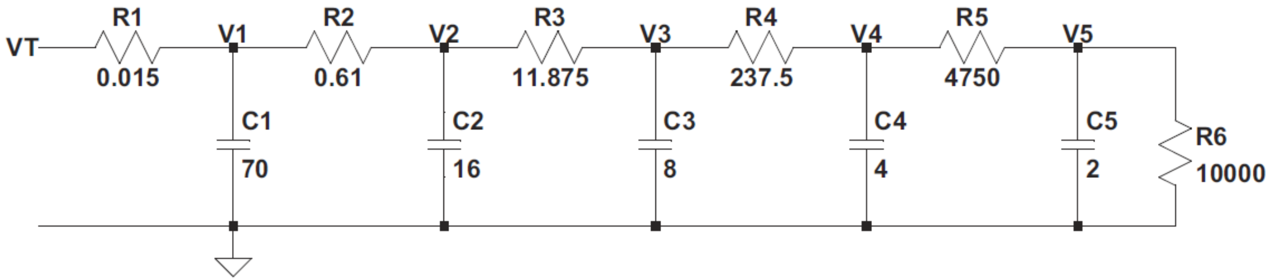
Peukert Constant Dependence on Temperature

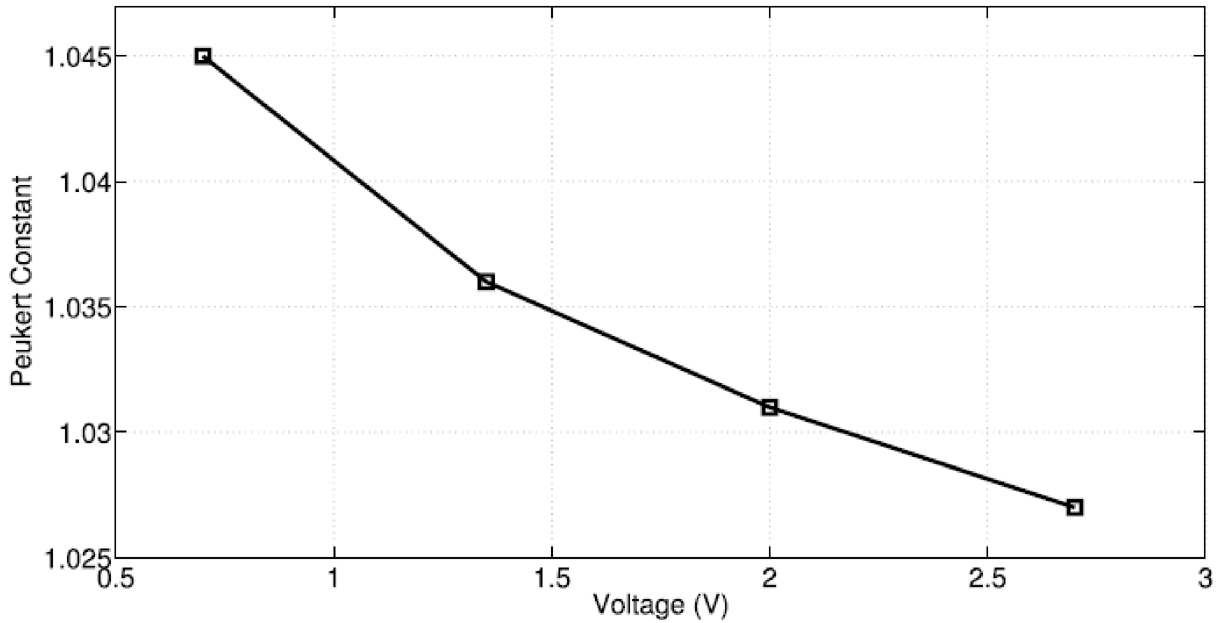
- Peukert constant increases when operating temperature is lower (initial voltage is 2.7 V and cutoff voltage is 0.01 V)



Simulation Results: Voltage Dependence

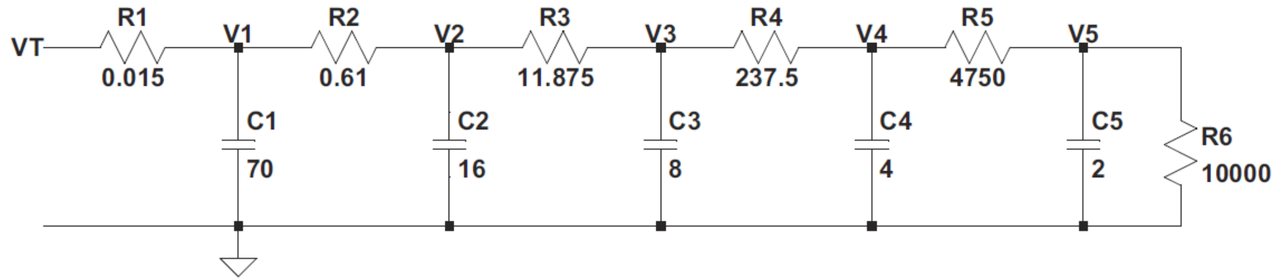
- Supercapacitor model
- Peukert constant dependence on initial voltage (cutoff voltage is 0.01 V)
- Effects of R_1 and C_1 on Peukert constant
  - k increases as R_1 increases
  - k increases as C_1 decreases

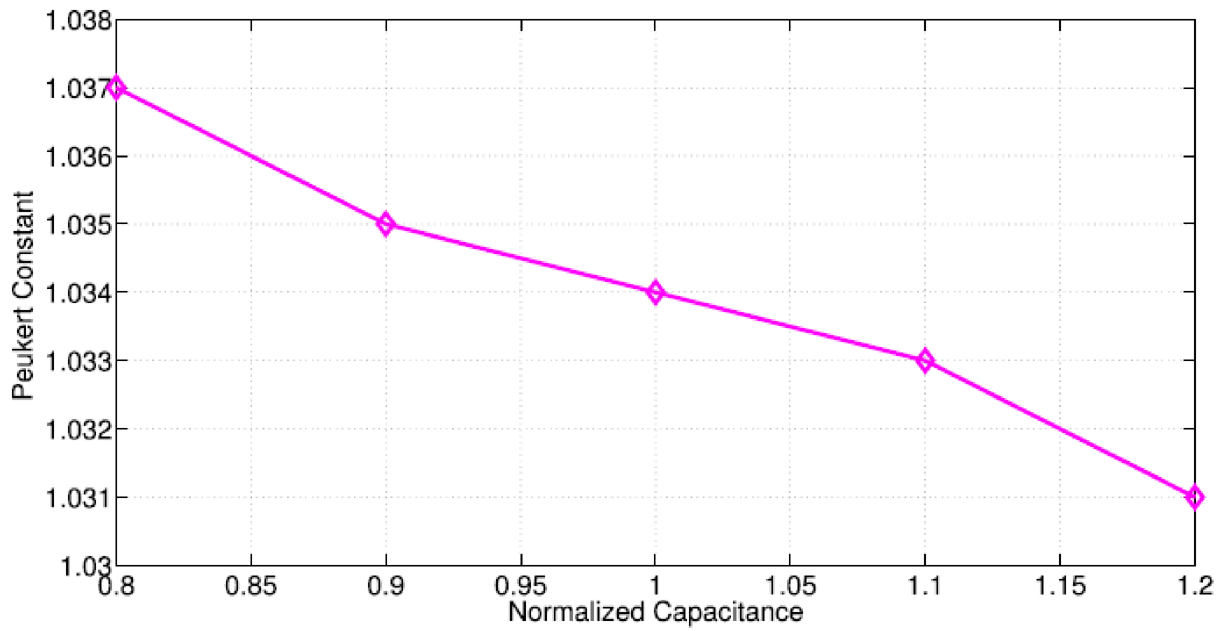




Simulation Results: Aging/Temperature Dependence

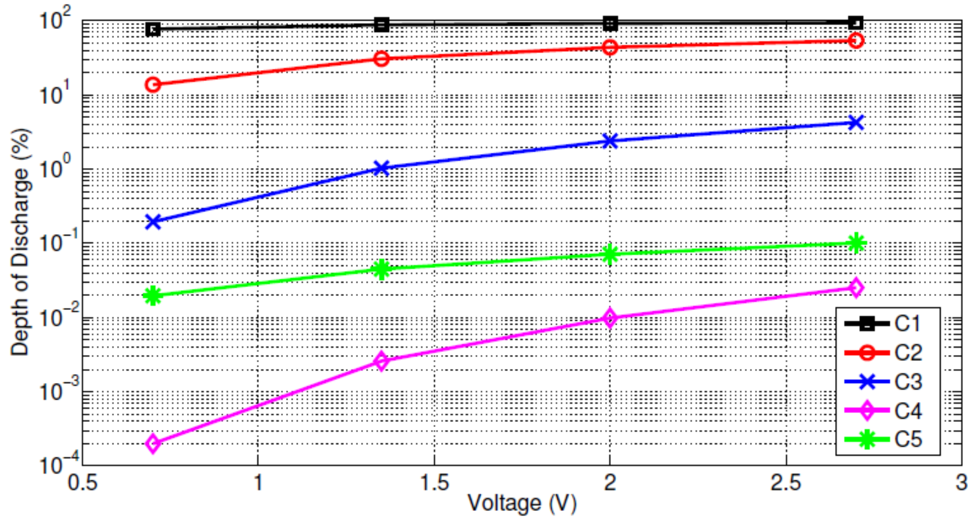
- Supercapacitor model
  - Upscaled: lighter aging or higher temperature results in larger capacitance and smaller resistance (e.g., 1.1)
  - Downscaled: smaller capacitance and larger resistance (e.g., 0.9)
- Peukert constant dependence on normalized capacitance

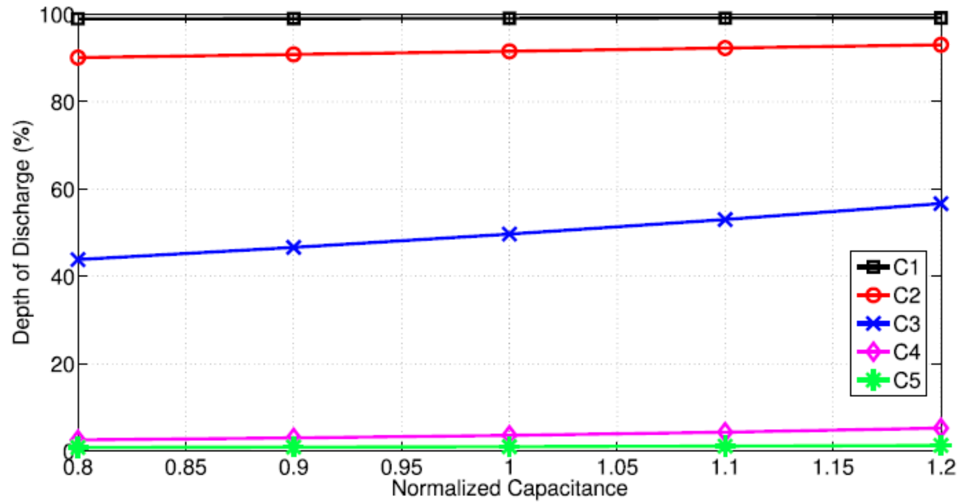




Physical Mechanisms: Effects of DOD on Peukert Constant

- Lower branch capacitor DOD (depth of discharge) results in larger Peukert constant: supercapacitor is less responsive or more relaxed
- Lower initial voltage (top): lower branch capacitor DOD
- Heavier aging/lower temperature (smaller normalized capacitance) (bottom): lower branch capacitor DOD





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- Research Interests
  - Supercapacitor Modeling and Characterization
  - Design and Control of Energy Storage Systems
  - Power Electronics for Energy Storage Applications
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