Lumped model to explain and approximate dimensional effects in ferrite cores

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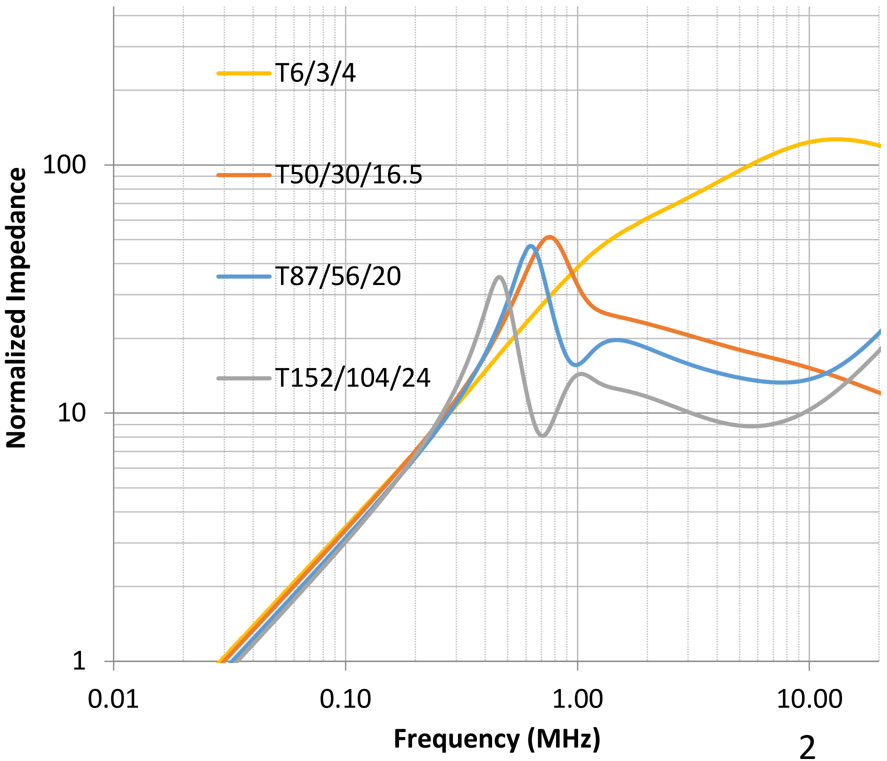
Core size affects magnetic material performance

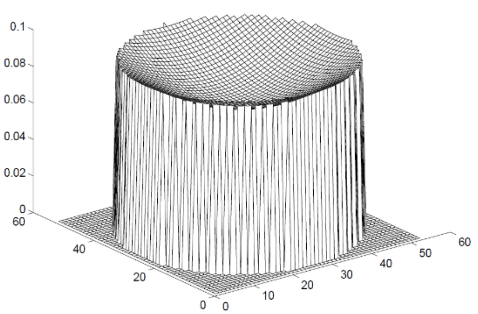
- A larger core has:
  - Higher losses in power applications.
  - Lower frequency rolloff in EMC applications.
- Two effects cause this:
  - Skin effect resulting from finite resistivity of ferrite.
  - “Dimensional resonance” effects: EM wave propagation is slow because of high ε_r and μ_r so interior can be out of sync with exterior.

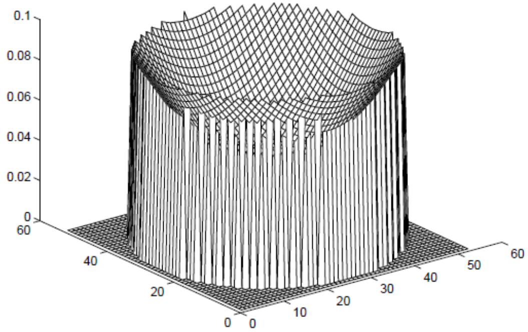
Four sizes of 3C95 MnZn ferrite toroids
(Marcin Kacki, SMA magnetics)



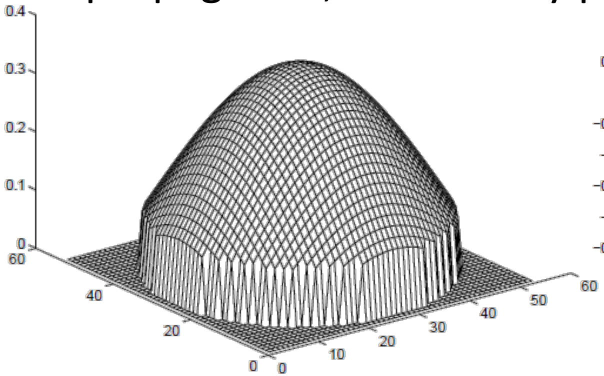
Examples of theoretical behavior: plots of |B| in a round centerpost

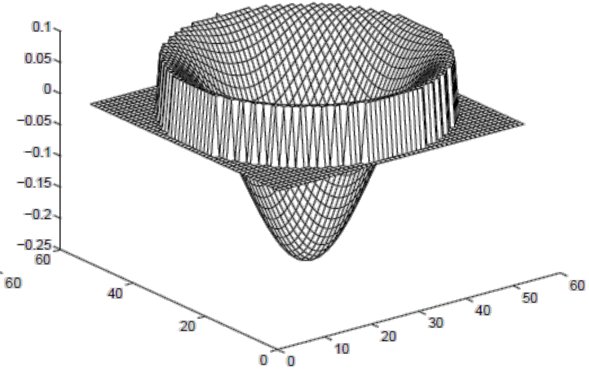
- Skin effect, affected by μ and σ (permeability and conductivity)





- Wave propagation, affected by μ (permeabilty) and ε (permittivity = dielectric const.)





- Figures from Glenn Skutt's excellent PhD thesis: "High-Frequency Dimensional Effects in Ferrite-Core Magnetic Devices," Virginia Tech, 1996.

How to predict these effects

- Need inherent permeability, permittivity, and conductivity of the material.
  - Permittivity and conductivity are not on typical datasheets.
  - Strong function of frequency.
  - Different for different materials.
- Can solve Maxwell’s equations if we assume linearity.
  - 2D FEA: most solvers don’t include both E and B in one solution.
  - 3D FEA is expensive in engineering time, computation time, and software licensing.
  - Analytical: possible for some cases.

Proposed alternative

- A lumped, ladder network approximation to the distributed behavior of the real system.
- This has been done before to model eddy currents in cores.
  - Also similar to standard models in many other applications, e.g. transmission line models.
- Now we include E field and displacement behavior as well.

Why this approach?

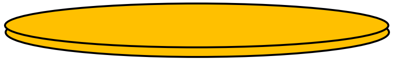
Why not a standard numerical solution to the partial differential equation, FEA or otherwise, in 1D or 2D?

- This is another option; not necessarily better.
- The model is accessible to electrical engineers:
  - The model structure can help provide intuition.
  - It can be solved by standard circuit simulators.
- The model can be incorporated into a simulation of a higher-order system, with or without model order reduction.
- Natural routes to extend to a nonlinear model.

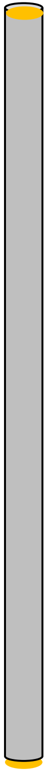
An intuition pump

- ε_r can be on the order of 100,000 (10^5)

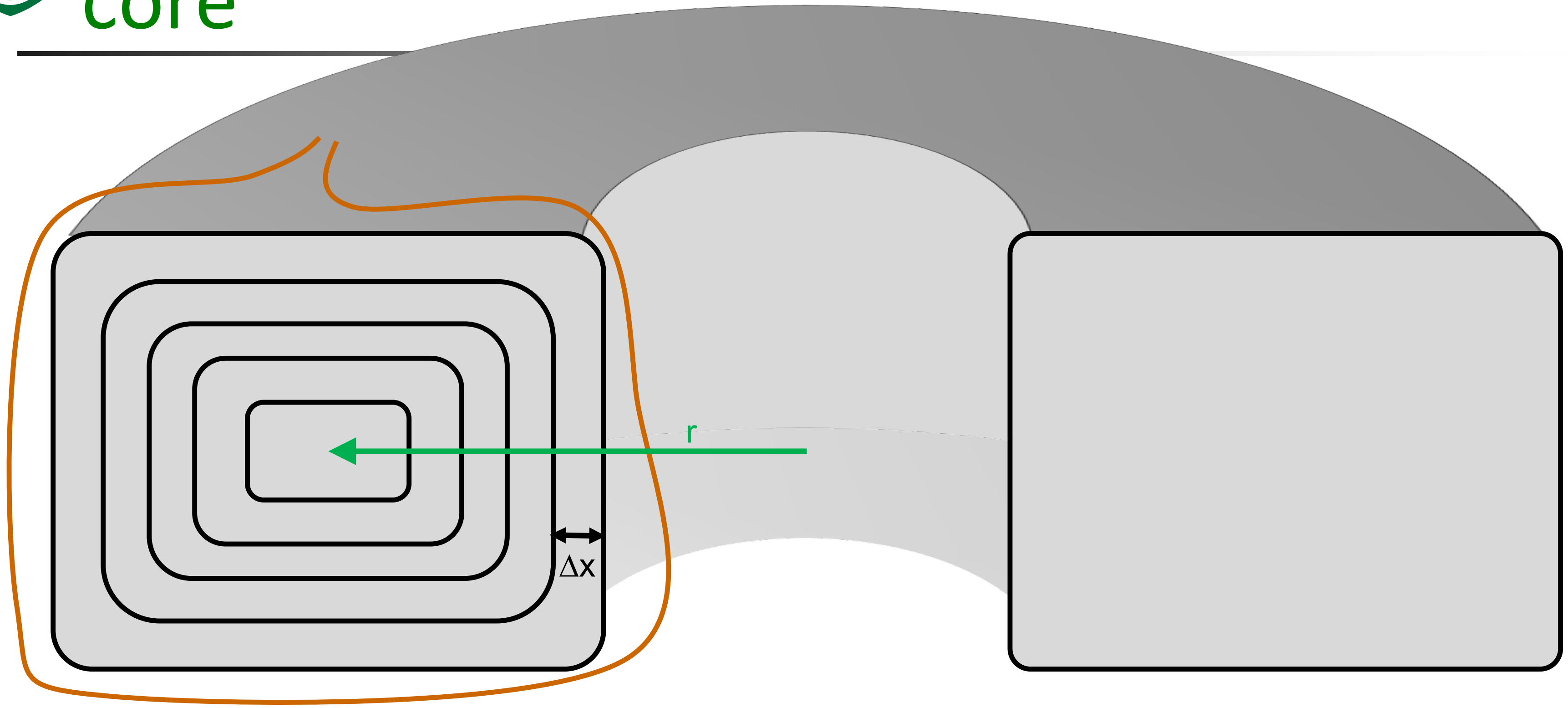
- An air capacitor with 50 mm diameter plates, 1 mm apart, has ~ 17.5 pF.



- With $\varepsilon_r = 10^5$, a 5 mm diameter rod, 100 mm long, metalized *at the ends* has 175 pF.

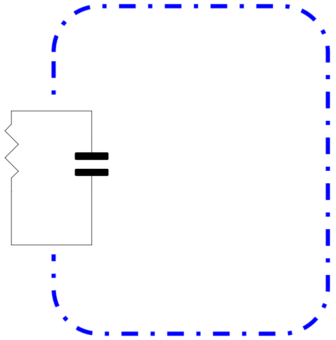


Consider concentric shells of a toroidal core



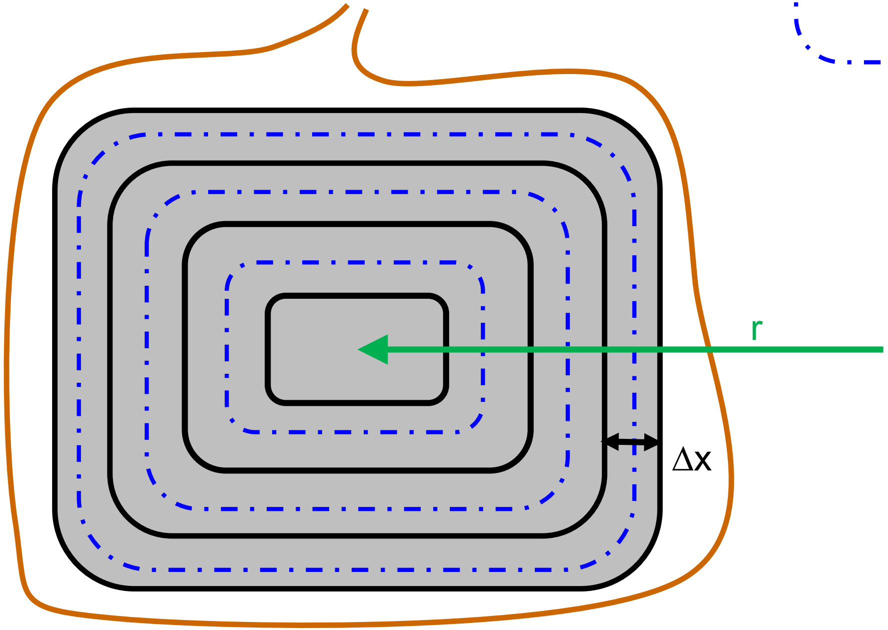
Broken up conceptually, but it's still solid material)

Basis of lumped model



- Each path (blue dashed line) has capacitance and resistance, in parallel:
  - $R = \ell\rho/A$
  - $C = \epsilon_r\epsilon_0 A/\ \ell$

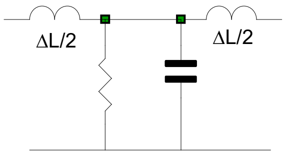
  where is the length of that loop, and A is its cross section, $A \approx (2\pi r)\Delta x$.

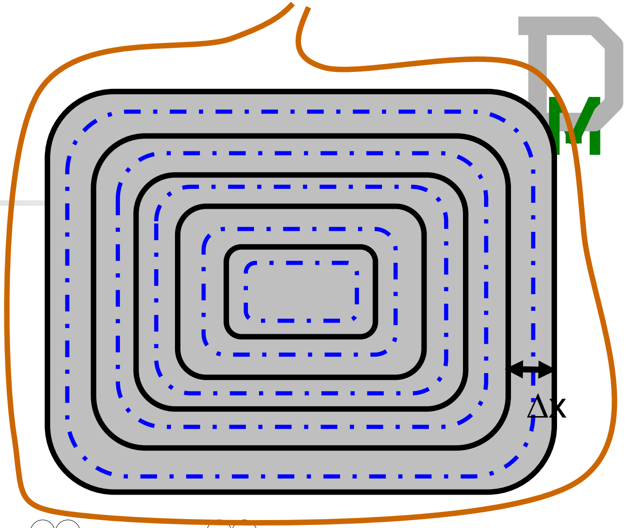


- Each path links a little less flux, as we move from the outer shells to the inner shells.
- The inductance arising from a shell divides half and half: half outside the blue line (not linked) and half inside (linked)
- Inductance of a shell
  $\Delta L = N^2\, A/(2\pi r) = (\Delta x\ \ell)\ /(2\pi r)$
  (for simplicity assume N =1)

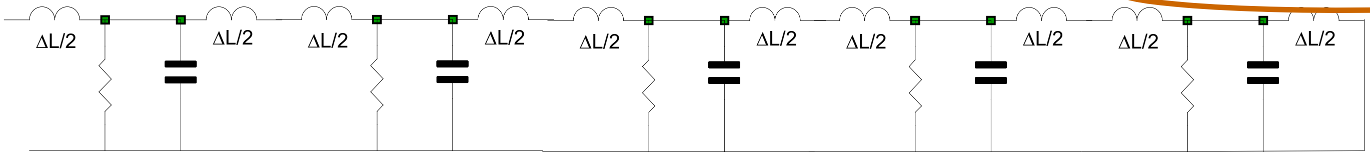
Full lumped model

- Model for one shell





- Transmission line model for multiple shells

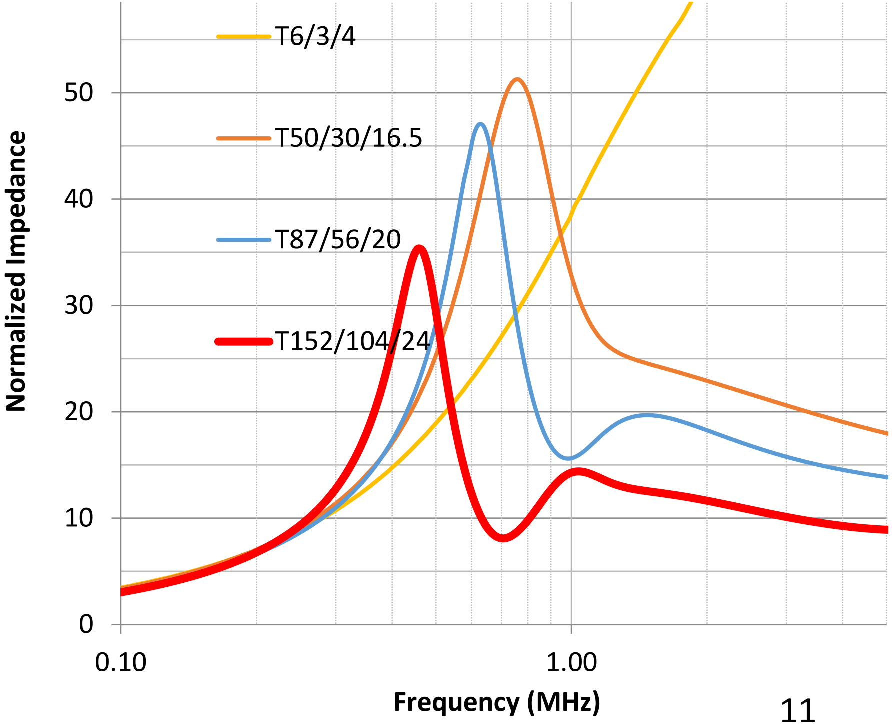


- This is a linear model, but any and all of the components can be made nonlinear and accurately capture nonlinear aspects of the corresponding behavior.
- Add an ideal transformer at the input for N > 1.

Can the lumped model predict this effect?

- We will use measured data for μ*(f), ε*(f) (complex permeability and complex permittivity).
- Complex permeability is derived from measurement of small toroid.
- Complex permittivity is derived from measurements of disk samples.

Four sizes of 3C95 MnZn ferrite toroids



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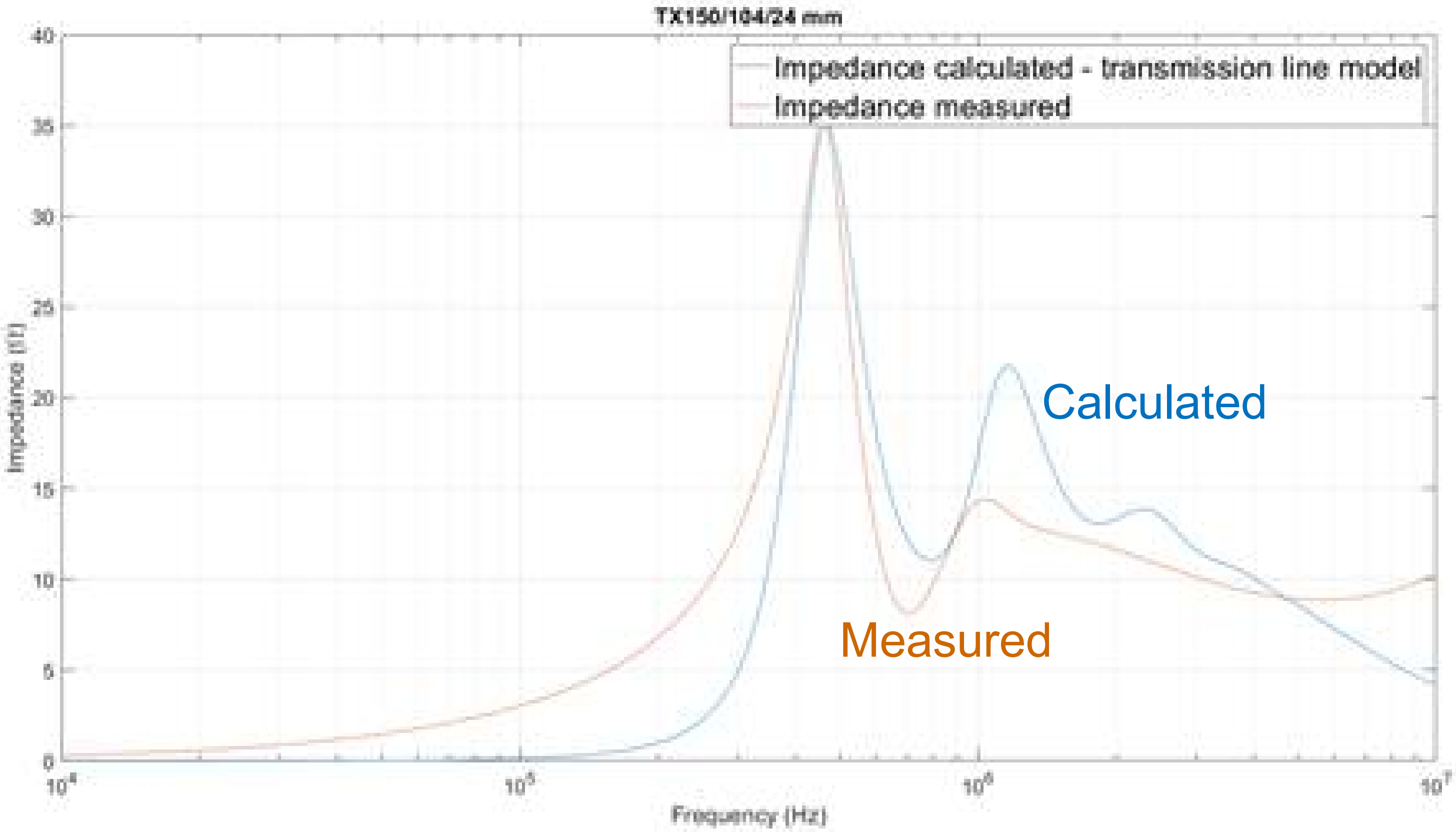
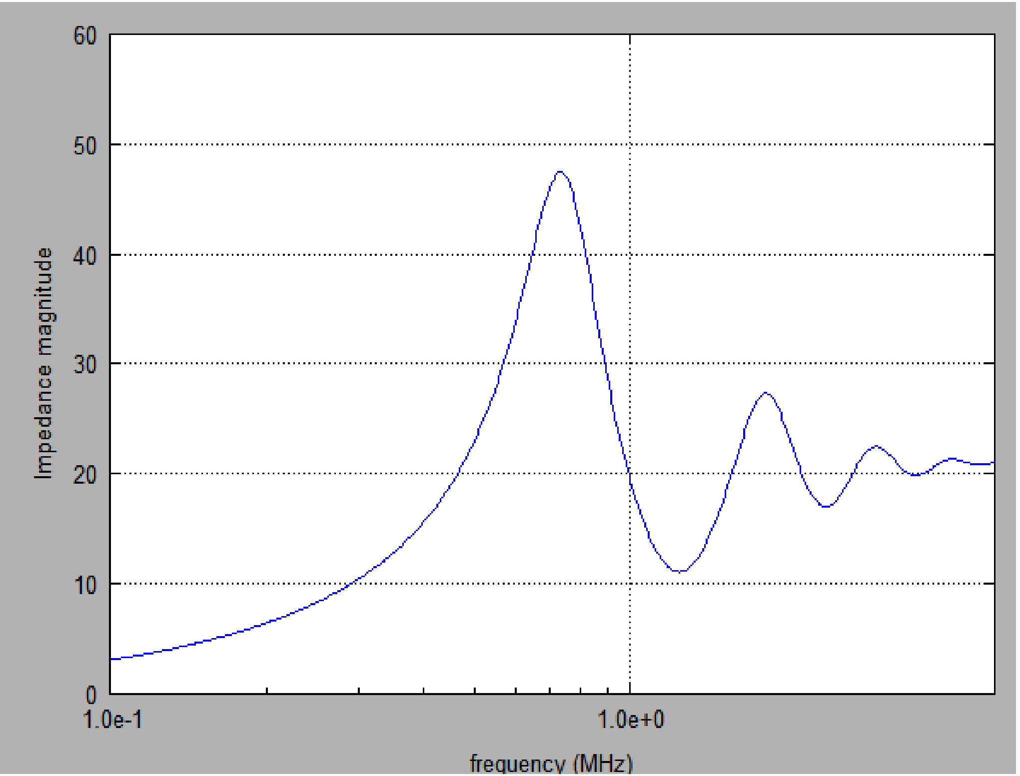
Good match to key features



Analysis and figure: Marcin Kacki, SMA magnetics

My results



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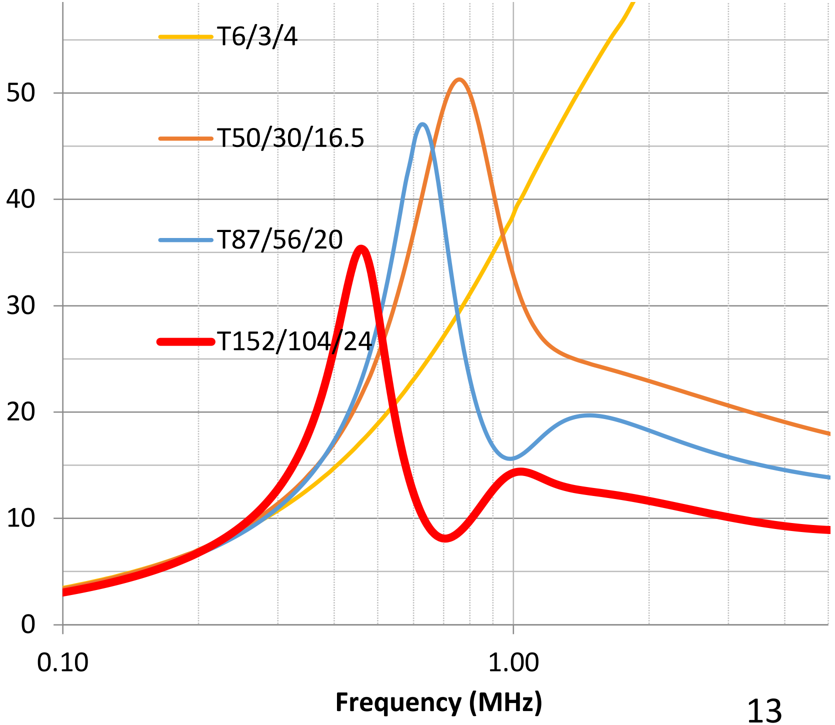
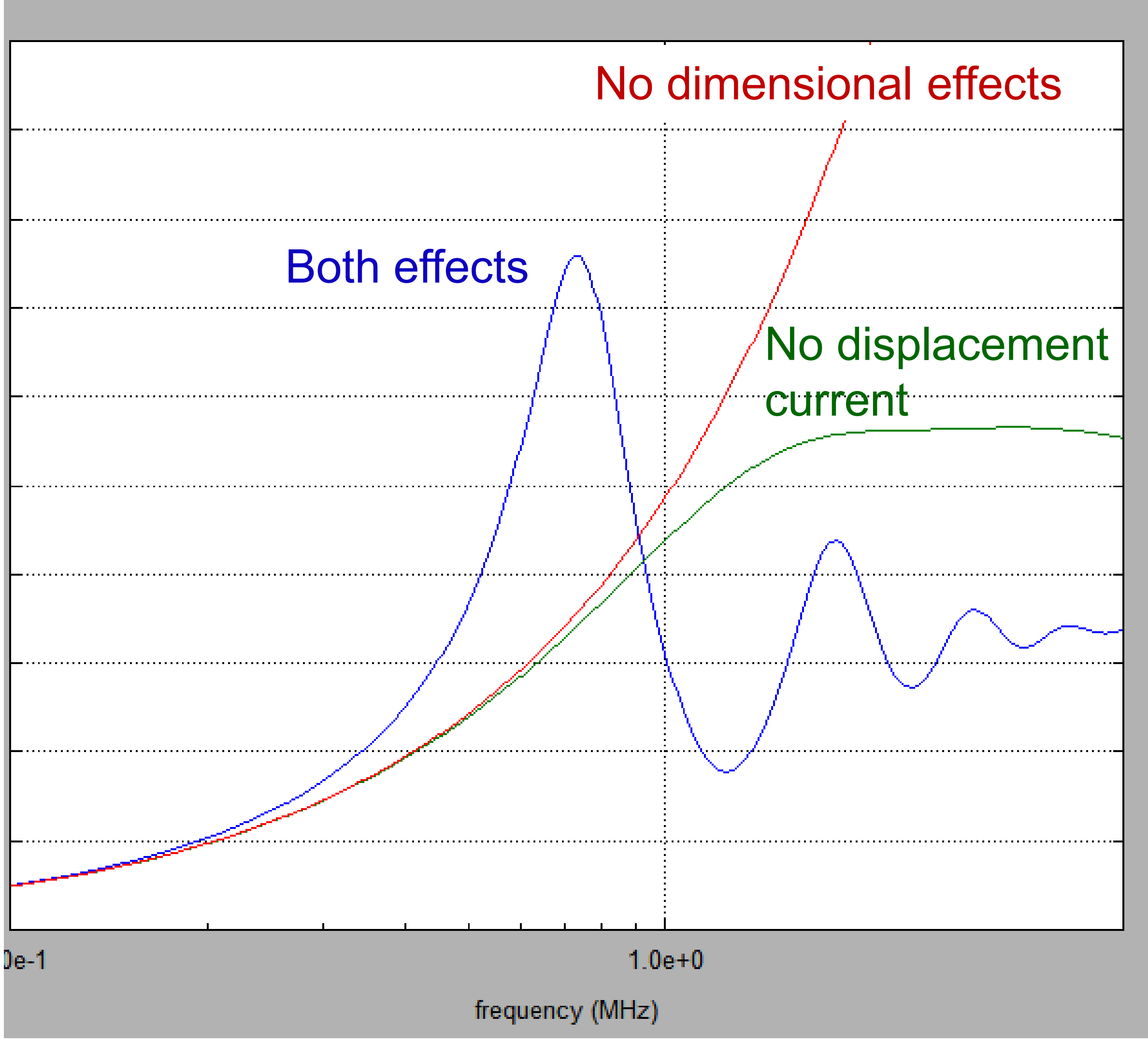


Individual effects

Conductive and displacement currents both matter.



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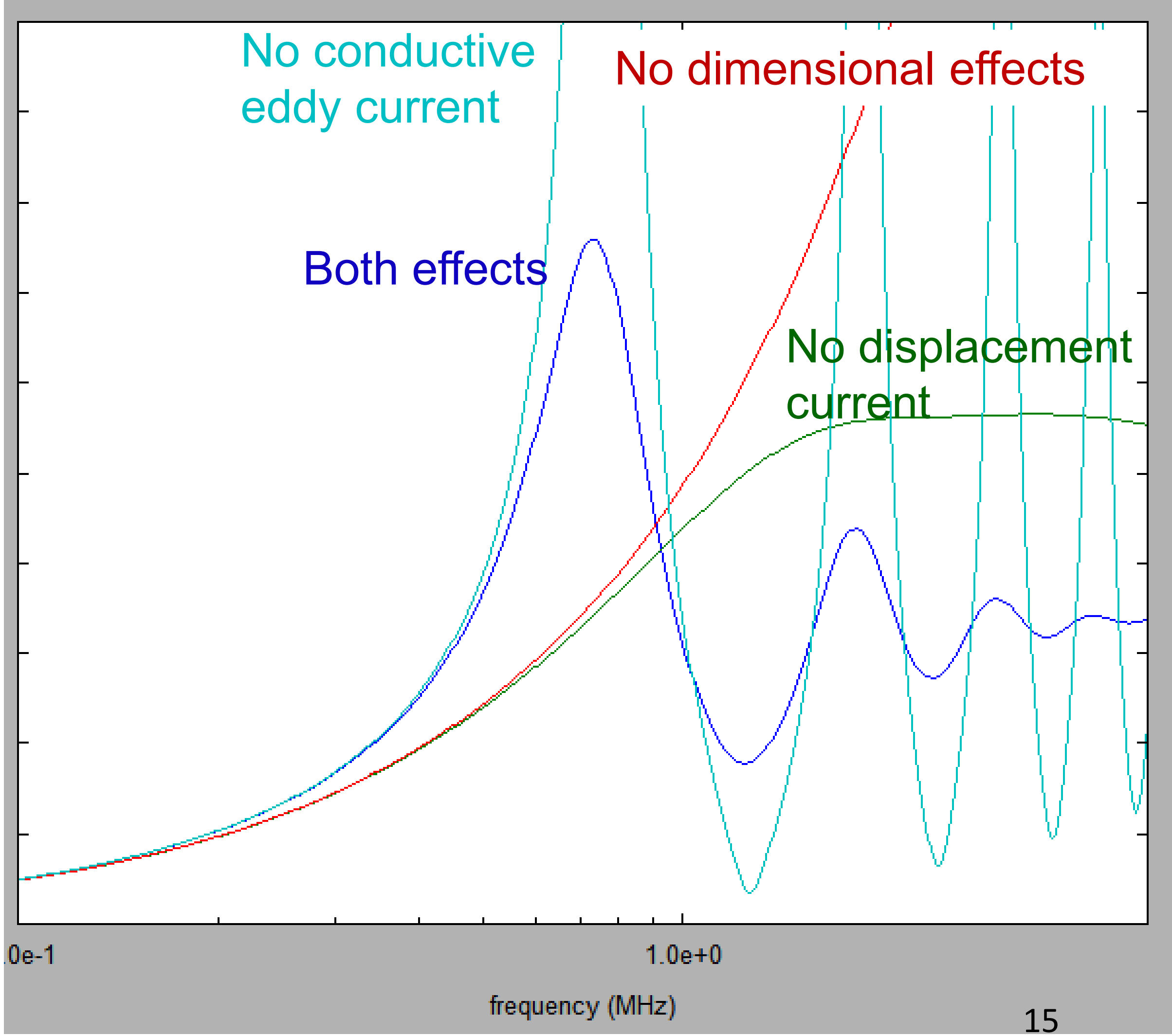
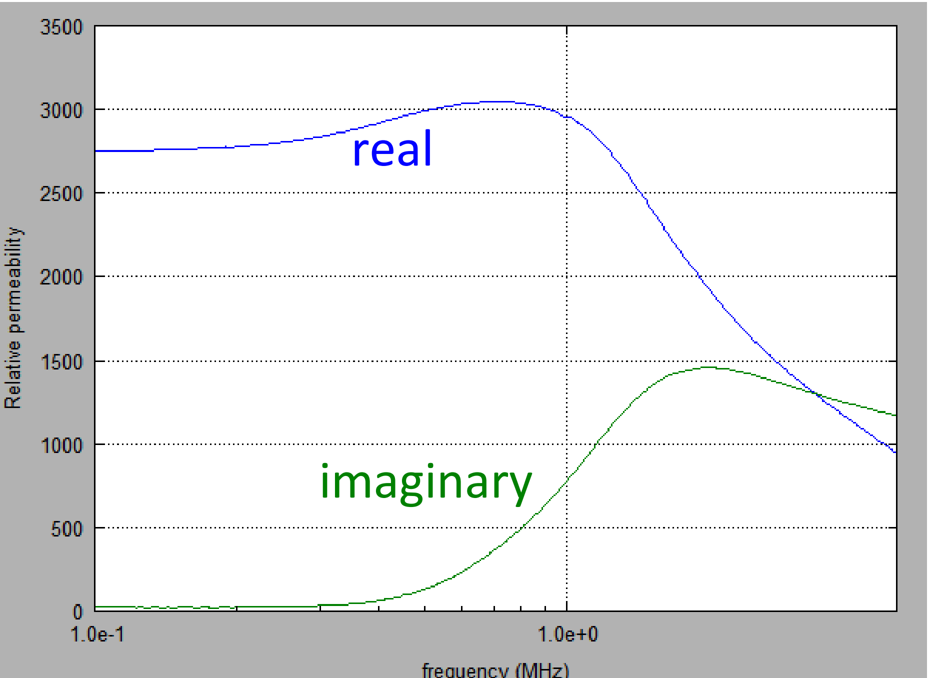
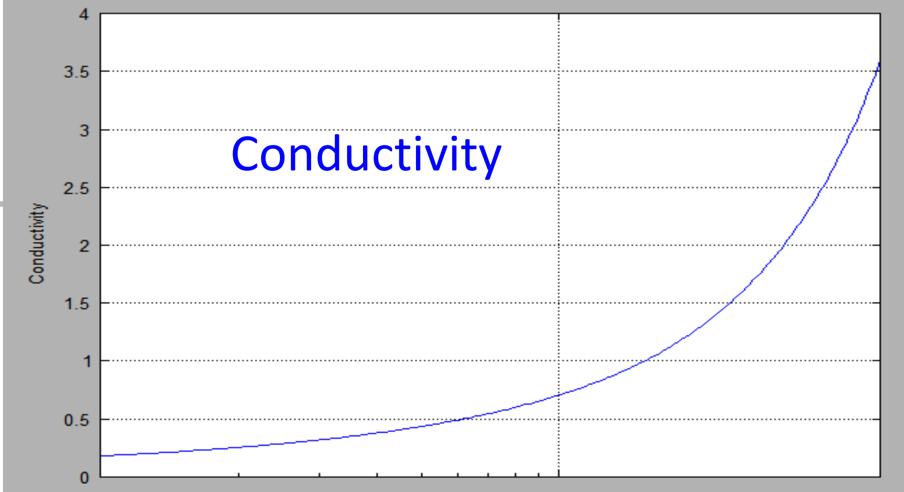
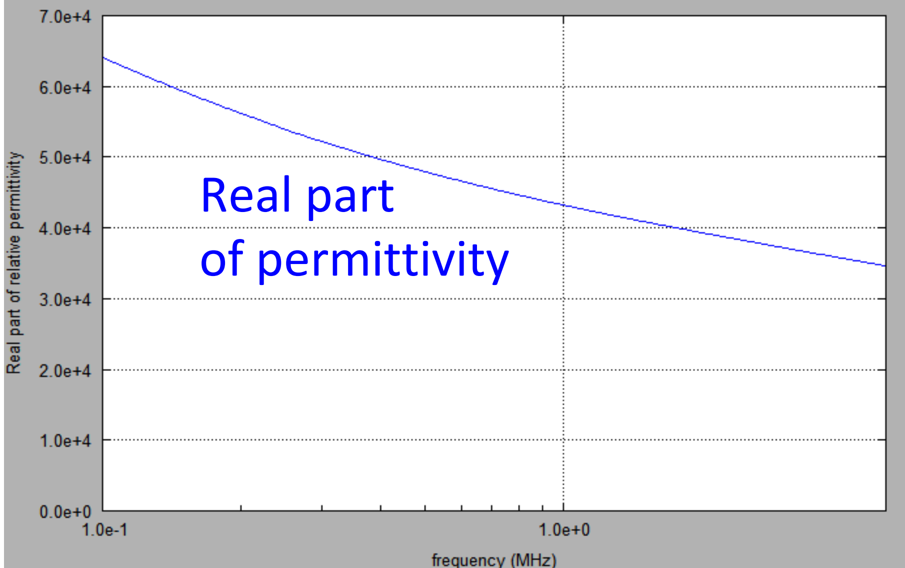


Variation of intrinsic properties w/ frequency

Complex Permeability

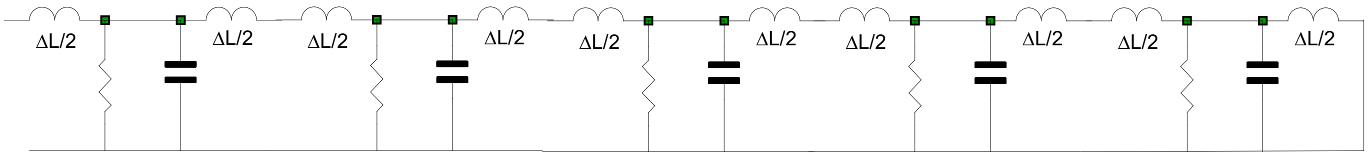






Is this model useful?

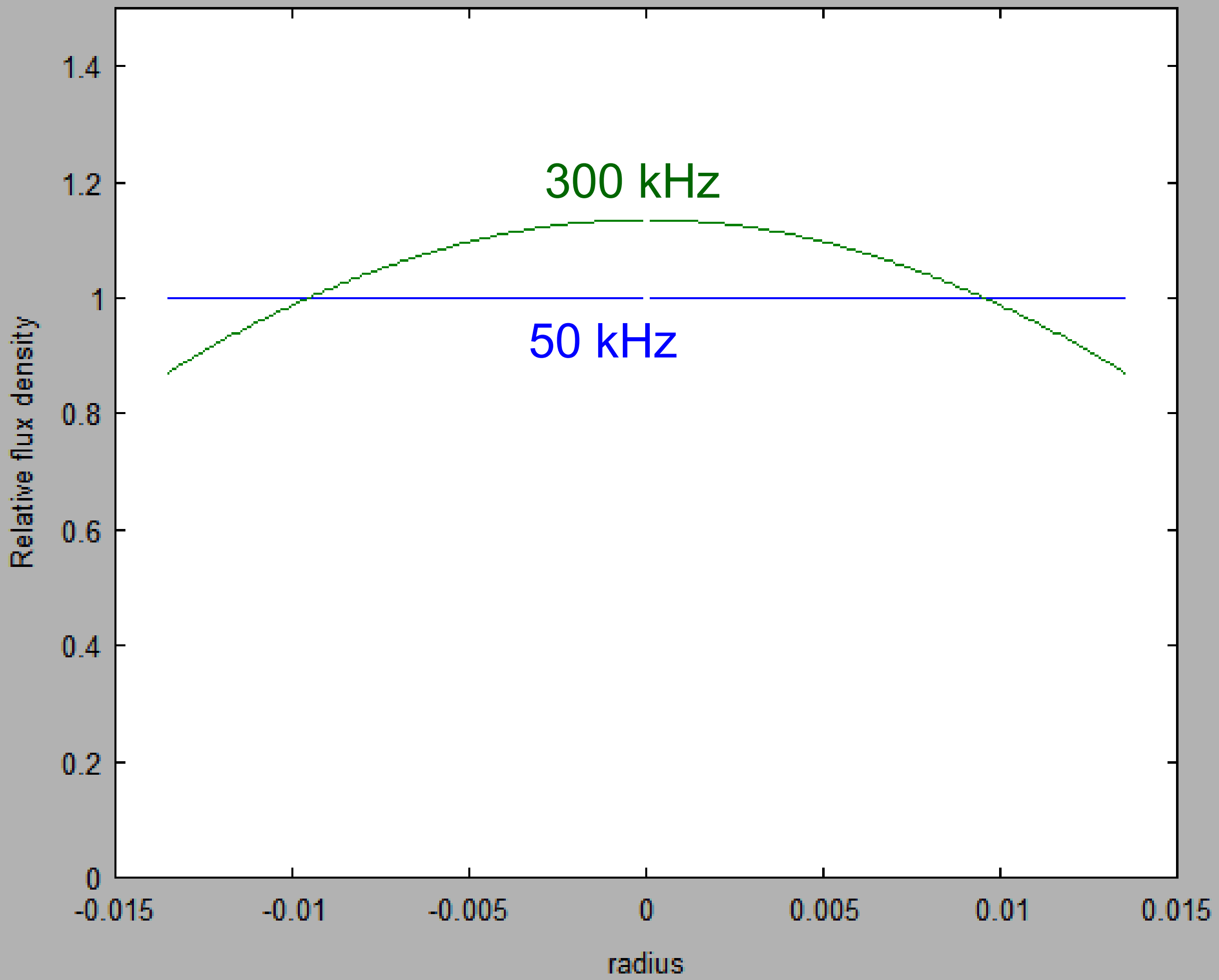
- Requires data on complex permeability and permittivity vs. frequency.
- These parameters also vary as a function of temperature.
- Doesn’t yet include nonlinearity or hysteresis—but these can be included by using nonlinear elements in the ladder.
- About 15 “rungs” in the ladder network is sufficient.
- Model as structured here works only at one frequency. For a wideband simulation model, replace each branch with a ladder network—the model becomes a ladder of ladders.
  - Model order reduction techniques could match the behavior for faster simulations.



Data requirements: for each temperature:

  - Complex permeability and complex permittivity of small samples over frequency: Two measurements.
  - Alternative: One measurement of effective complex permeability for each core size.
- Conclusion: fewer measurements needed to address a wide range of core sizes.

Visualize flux density vs. radius



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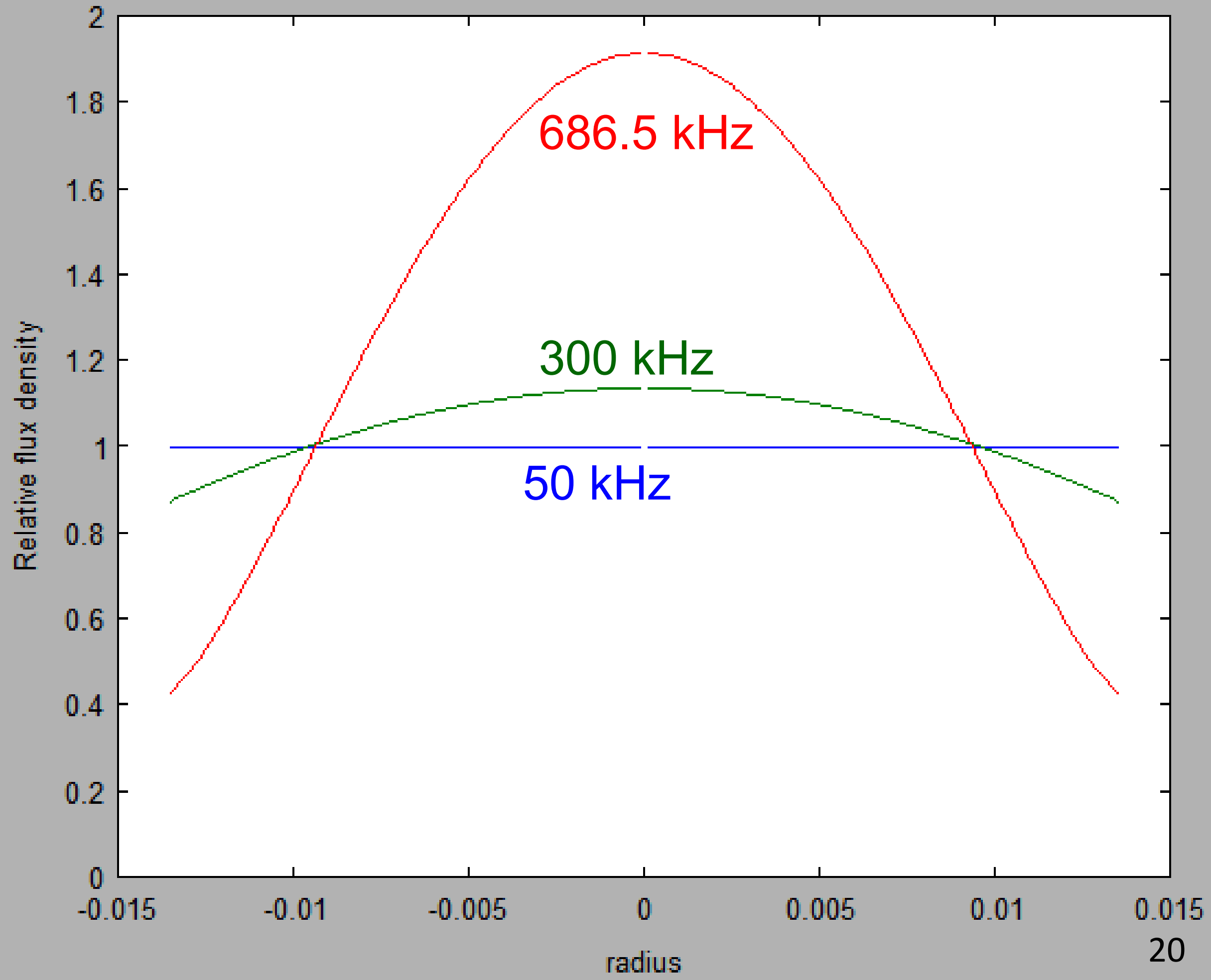

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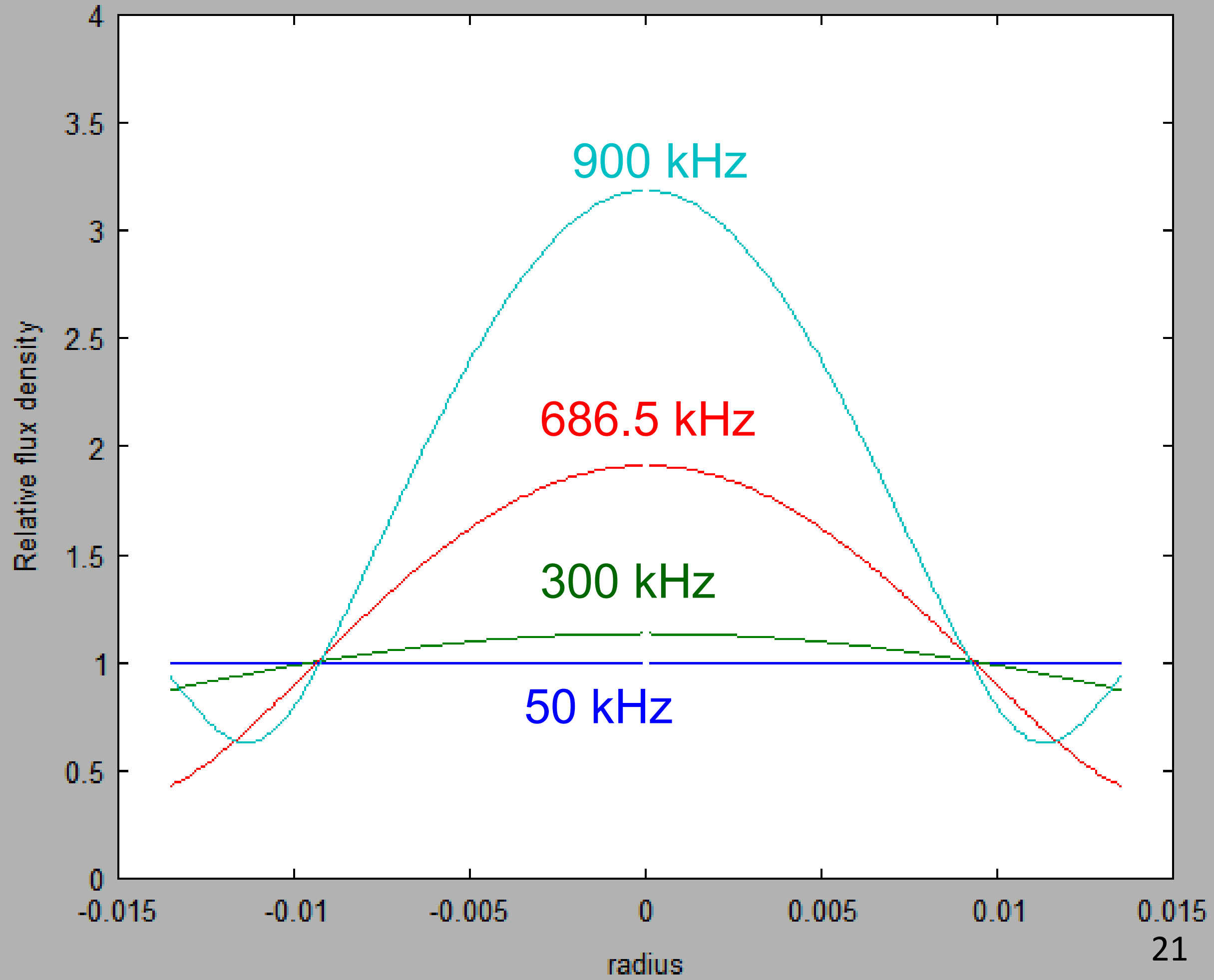

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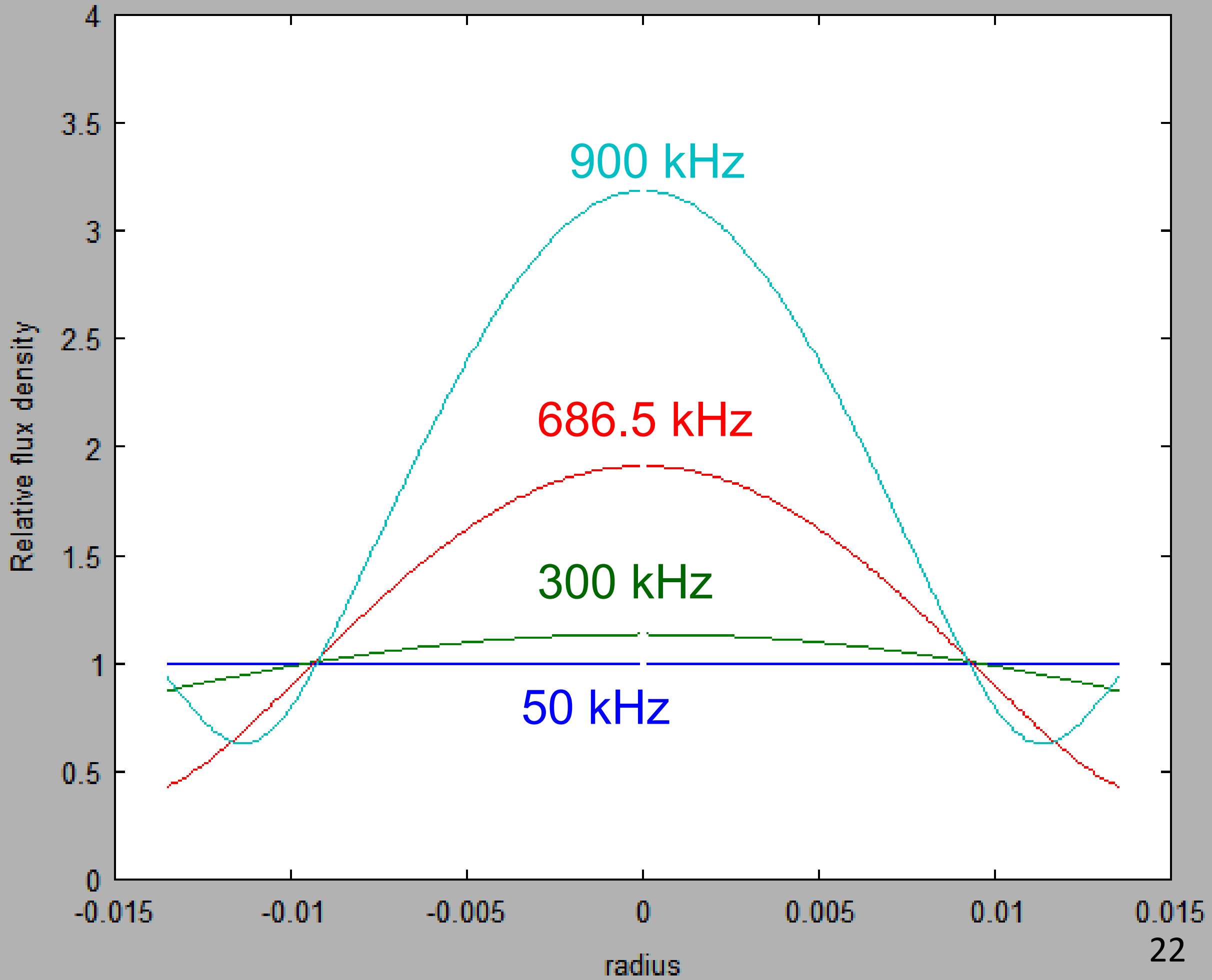

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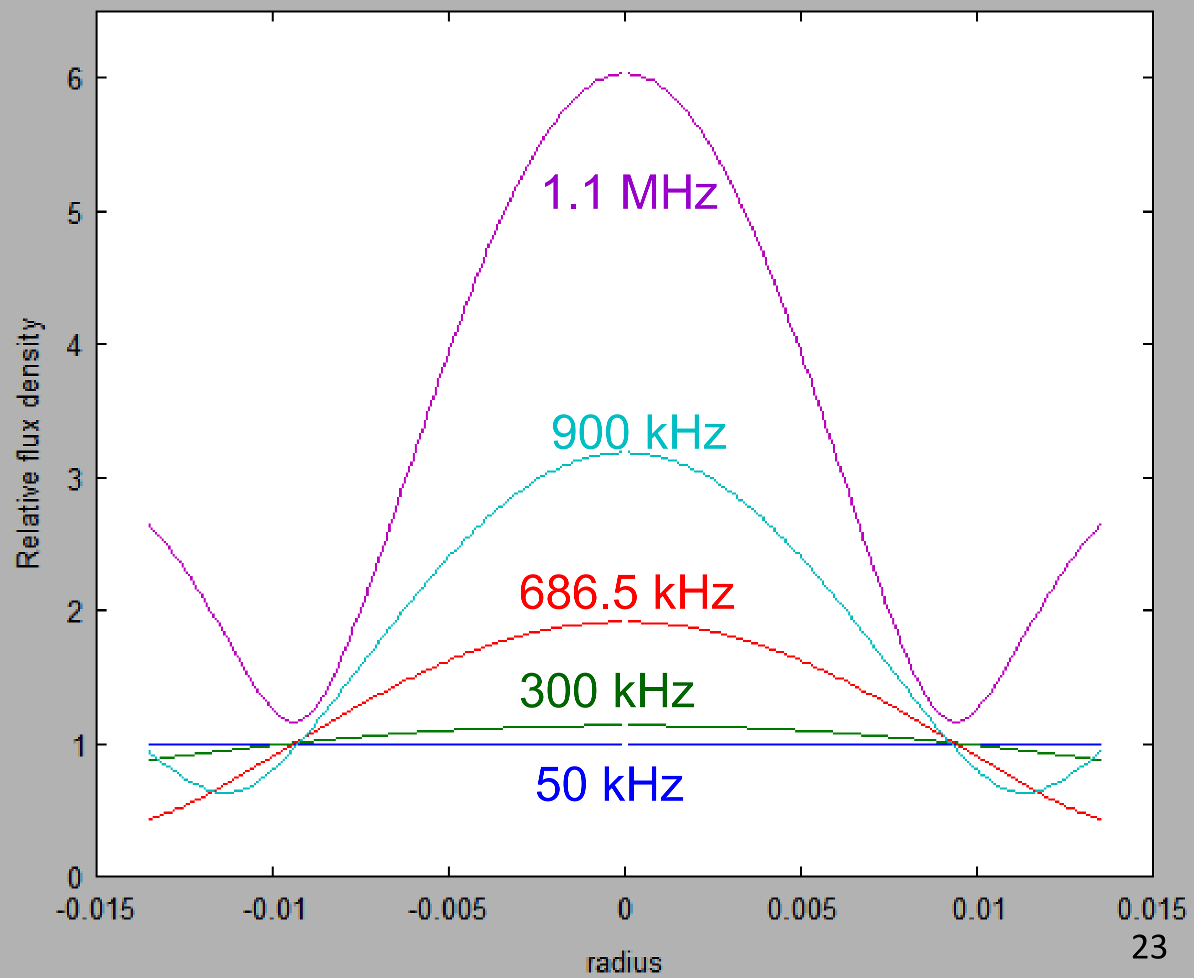
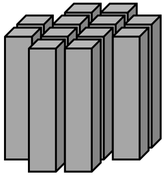


Solutions

- Breaking up core into small sticks or sheets can avoid these problems.
  
  - Same concept as steel laminations for line frequency, but addresses displacement current as well as conduction current.
  - Avoids the need for the complex model as well as reducing losses and extending frequency range of a filter.

References on similar models without displacement current

- Basic model topology has been used before for conduction eddy currents, e.g. for laminations in these references:
  - IEEE TRANSACTIONS ON MAGNETICS, VOL. 33, NO. 2, MARCH 1997, Modelling eddy currents and hysteresis in a transformer laminate, P. Holmberg, A. Bergqvist, G. Engdahl, https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=582495
  - C. R. Sullivan and A. Muetze, "Simulation Model of Common-Mode Chokes for High-Power Applications," in *IEEE Transactions on Industry Applications*, vol. 46, no. 2, pp. 884-891, March-april 2010. URL: http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=5406120&isnumber=5433727