Core Shape Effects

Prof. Charles R. Sullivan and Dr. Thomas P. H. Guillod

chrs@dartmouth.edu

PMIC *Power Management Integration Center*

Dartmouth Magnetics and Power Electronics Research Group

THAYER SCHOOL OF ENGINEERING AT DARTMOUTH

http://dartgo.org/pmic

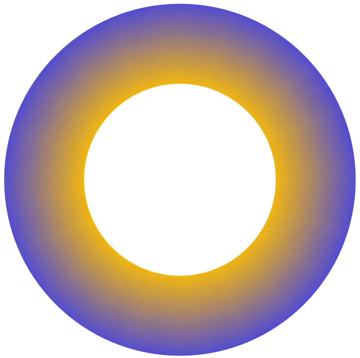
Effects of Core Shape

Focus today: ferrite.
Some issues are similar for other materials; some are different.

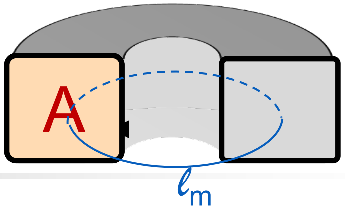
- Effects on core loss:
  - Static linear
  - Static nonlinear
  - Dynamic
- Effects on core fabrication process and material properties.
- Effects on the field in the winding region:
  - Leakage inductance
  - Winding loss
- Thermal: cooling capability
- Component considerations:
  - Winding fabrication considerations
  - Design parameters: winding length and the winding area/core area ratio.

Flux distribution: linear, static case

- Toroid
- Constant permeability μ_r
- Flux density higher at inner radius.
- What's the effect on loss?
  - Compared to what?
  - Compared to calculation based on core area A and core length ℓ.



What area and length?



- Actual area and mean length don't work well:
  - Assume loss $P \propto B^{\beta}$, $\quad 2 < \beta < 3$
  - For radius ratio $r_o/r_i = 3$,
    - Loss is up to 30% worse than predicted (at $\beta = 3$)*
    - But inductance isn't predicted well either: motivation to use effective area and length A_e, ℓ_e

*See appendix for details of when it's worst.

Effective area and length

- Chosen for exact inductance.
- As given in datasheets
- To evaluate how good these are, we again:
  - Assume loss $P \propto B^{\beta}$, $\quad 2 < \beta < 3$
  - For radius ratio $r_o/r_i = 3$

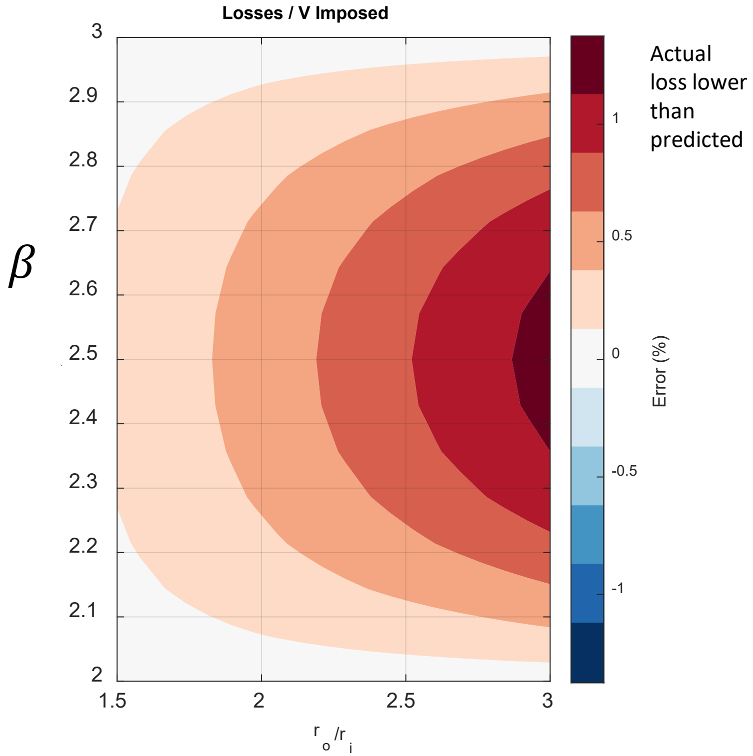
$$l_e = \frac{2\pi \ln\left(\frac{r_o}{r_i}\right)}{\frac{1}{r_i} - \frac{1}{r_o}}$$

$$A_e = \frac{h \ln^2\left(\frac{r_o}{r_i}\right)}{\frac{1}{r_i} - \frac{1}{r_o}}$$

IEC, "Calculation of the Effective Parameters of Magnetic Piece Parts (2nd ed.,). IEC 60205", 2001

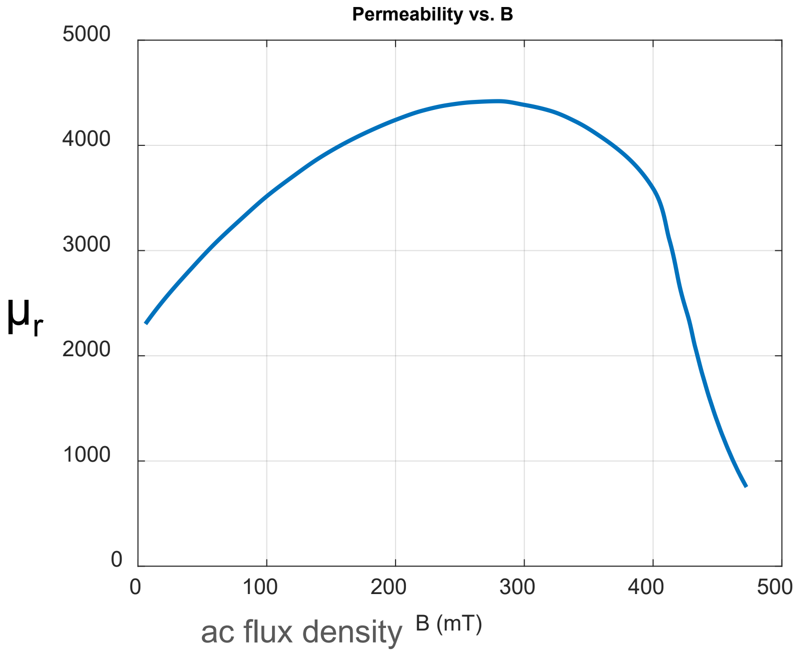
Effective area and length

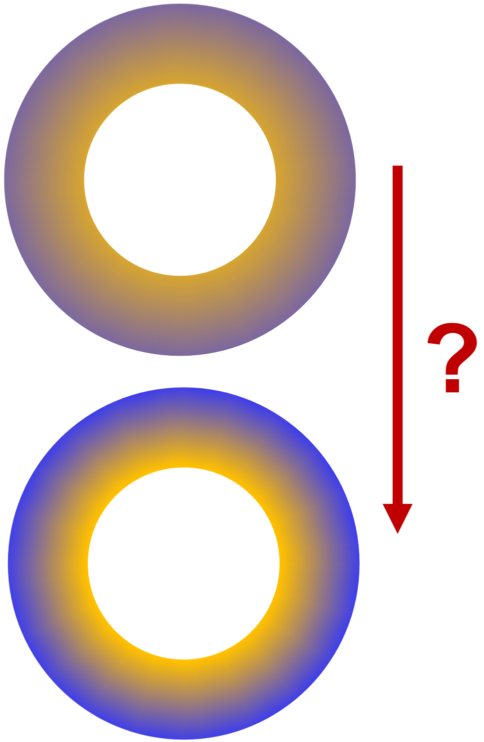
- Error in power loss is very small:
  - Worst case is 1.2%.
  - Zero error for $\beta = 2$ ***or*** $\beta = 3$
- Using the standard effective parameters works very well over the full range of β values.
- Also gives an exact prediction of inductance, and so the loss results are the same for voltage or current imposed.



Nonlinear effects on static flux distribution.

Variation in large-signal ac permeability can magnify the non-uniformity of the flux distribution.

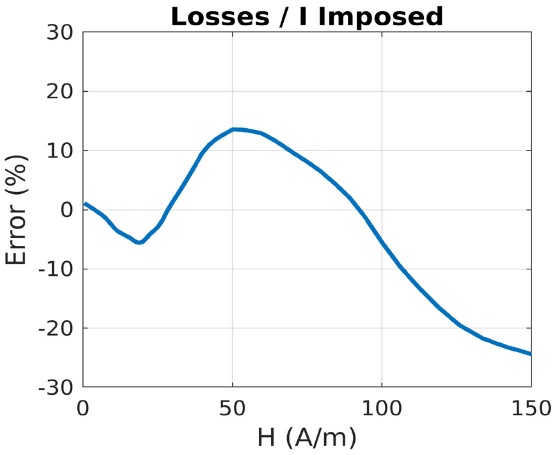


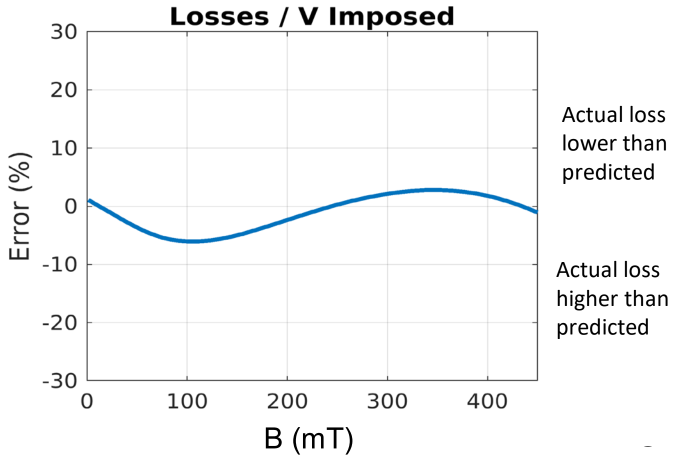


Nonlinear effects on static flux distribution.

Example results:

- N87 ferrite, $r_o/r_i = 3$
- Direction of impact depends on amplitude.
- Much larger impact than in the linear case, but not catastrophic.
- Inductance also varies—this explains why the impact depends on whether you fix current or fix voltage (or equivalently, flux). More plots in the appendix.

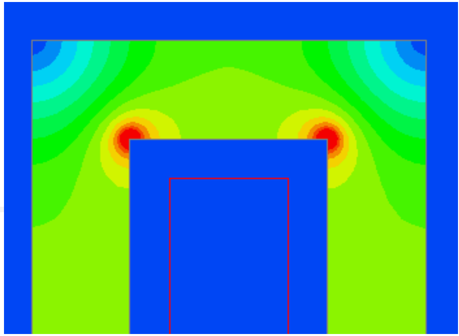




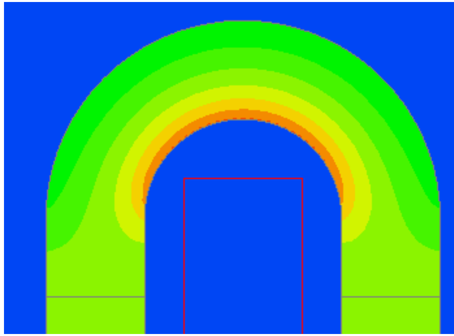
More complex core shape

- Crowding is primarily at corners:
  - More dramatic concentration of flux—the corner will often saturate leading to high loss density.
  - But the high loss density is only in a small volume—overall impact on loss is not severe.
- May require 2- or 3-D FEA analysis for more accurate exact assessment.

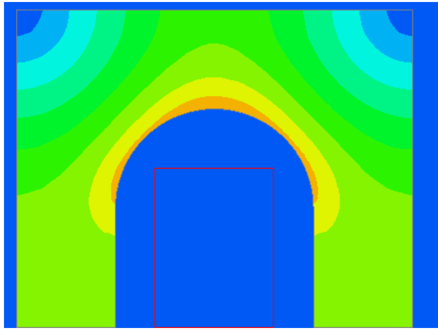
+3% loss



Base case

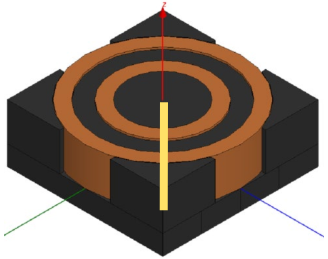


-5% loss

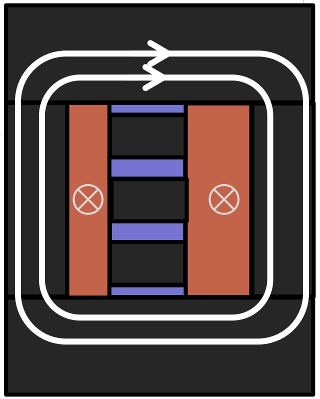


More complex core shape

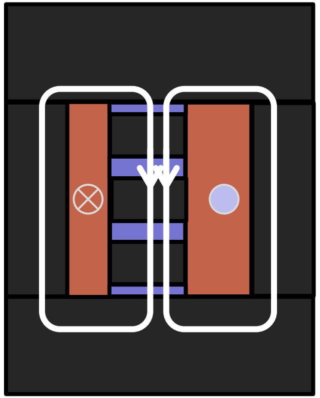
- Crowding is primarily at corners:
  - More dramatic concentration of flux.
  - Loss increase is only in a small volume.
- May require 2- or 3-D FEA analysis for more accurate exact assessment.
  - Example results: 16.7% increase in loss vs. simple reluctance model.



Magnetizing Excitation



Leakage Excitation



Excitation	Loss (arbitrary units)		
	FEA	Model	Difference
Leakage	1514.74	1331.39	12.10%
Magnetizing	10602.43	8834.1	16.68%

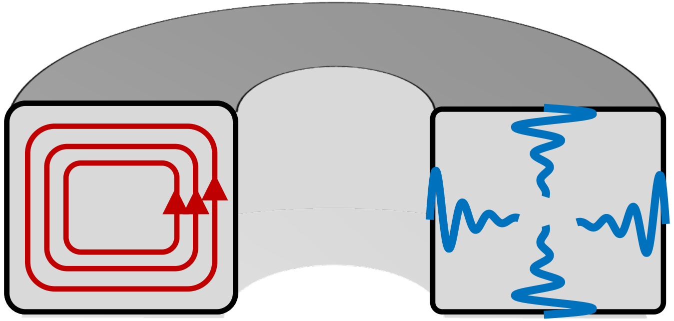
G. E. Gamache and C. R. Sullivan, "Resonant converter transformer design and optimization," *IEEE ECCE*, 2011, pp. 590-597, doi: 10.1109/ECCE.2011.6063823.

Effects of Core Shape

- Effects on core loss:
  - Static linear
  - Static nonlinear
  - **Dynamic**
- Effects on core fabrication process and material properties.
- Effects on the field in the winding region:
  - Leakage inductance
  - Winding loss
- Thermal: cooling capability
- Component considerations:
  - Winding fabrication considerations
  - Design parameters: winding length and the winding area/core area ratio.

Dynamic dimensional effects on core loss

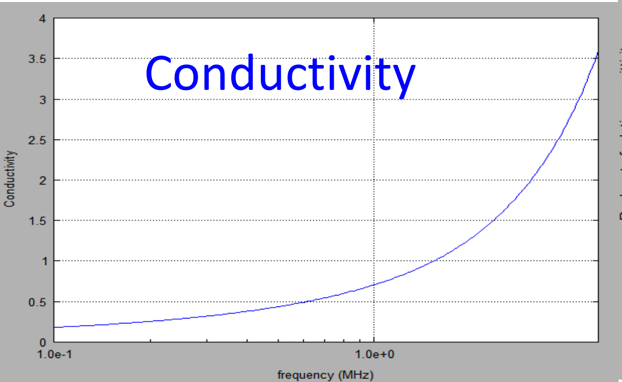
- Two effects:
  - Eddy currents: even ferrite has finite conductivity.
  - Wave propagation (a.k.a. dimensional resonance).
- Both result in:
  - Increased core loss at high frequency.
  - Decreased impedance at high frequency (concern for EMI chokes).
  - Worst for large cores:
    - Eddy current link more flux
    - Wave propagation has further to go.

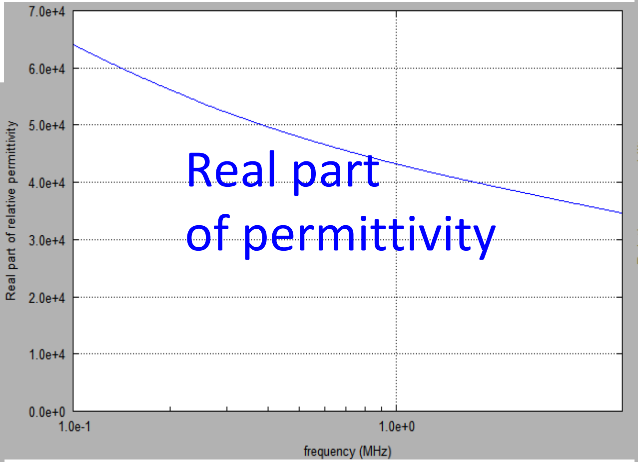


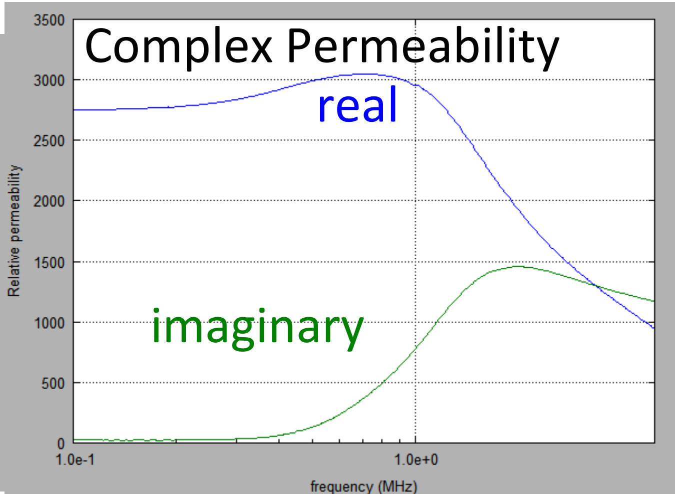
Parameters

- Eddy current depends on conductivity σ.
  - Skin depth $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$
- Wave propagation speed v depends on permeability μ and permittivity ε: $v = \frac{c}{\sqrt{\mu \varepsilon}}$
  - $\frac{\lambda}{4} = \frac{c}{4f\sqrt{\mu\varepsilon}}$

- All vary with frequency *and temperature* (factor of 10 on conductivity).
- Different for different materials (e.g., N87 vs. N49) and very different for MnZn vs. NiZn—part of how NiZn works at higher frequency.







Resulting skin depth and wavelength: circa 2 cm at 1 MHz in MnZn ferrite

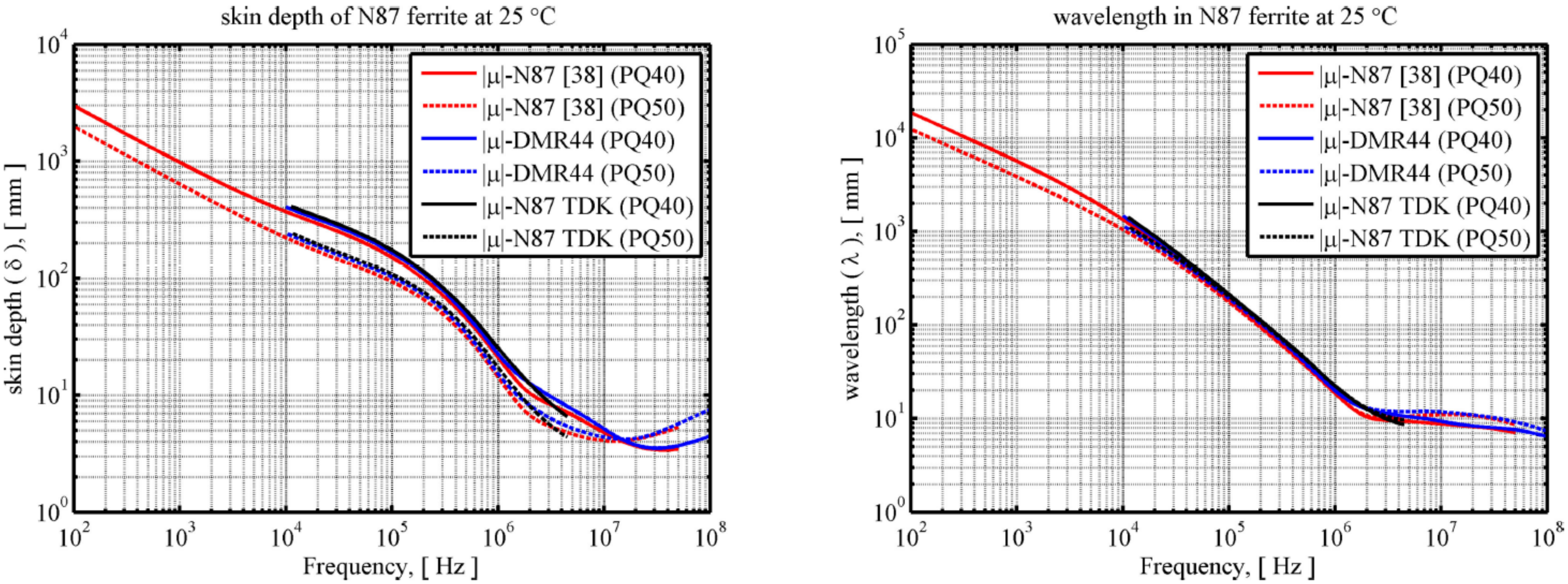


Fig. 11. Skin depth of and electromagnetic wavelength in N87 ferrite material, obtained by the extracted electrical properties and the three permeability cases plotted in Fig. 7.

* T. P. Todorova, A. Van den Bossche and V. C. Valchev, "A Procedure for the Extraction of Intrinsic AC Conductivity and Dielectric Constant of N87 Mn–Zn Ferrite Samples Based on Impedance Measurements and Equivalent Electrical Circuit Modeling," in *IEEE Trans. on Pow Electr.*, 33(12), Dec. 2018. doi: 10.1109/TPEL.2018.2802787

Simulations with wave propagation and skin effect

Ideal

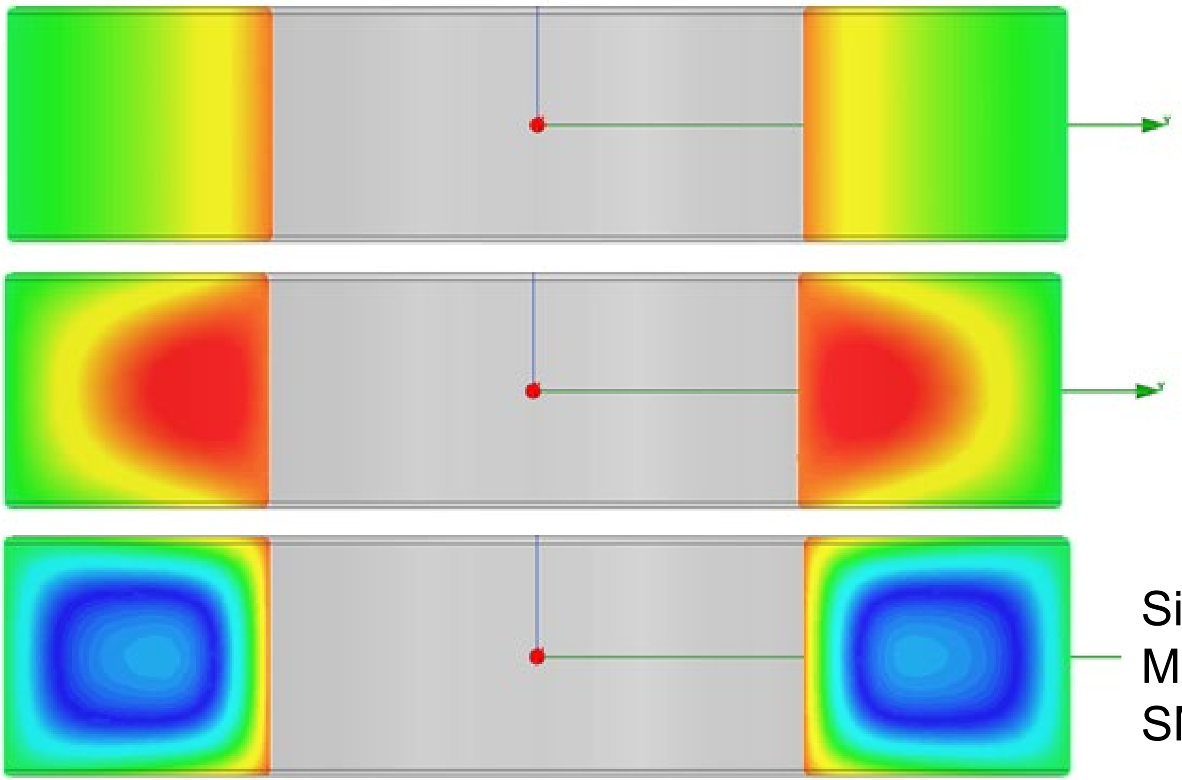
f = 500 kHz
μ = 10 000
ε = 0.1
ρ = 0.1 S/m

Wave

f = 500 kHz
μ = 10 000
ε = 20 000
ρ = 0.1 S/m

Both

f = 500 kHz
μ = 10 000
ε = 20 000
ρ = 5 S/m



Simulations by Marcin Kacki, SMA Magnetics

How to predict these effects

- Need inherent permeability, permittivity, and conductivity of the material.
  - Permittivity and conductivity are not on typical datasheets.
  - Strong function of frequency and temperature.
  - Different for different materials.
  - Measurements are tricky—on large samples dimensional effects skew the results.
- Can solve Maxwell's equations if we assume linearity.
  - 2D FEA: most solvers don't include both E and B in one solution.
  - 3D FEA is expensive in engineering time, computation time, and software licensing.
  - Analytical: possible for some cases:
    - 1-D, e.g. in a laminated core. See:
      Brockman, F.G., Dowling, P.H. and Steneck, W.G., "Dimensional effects resulting from a high dielectric constant found in a ferromagnetic ferrite". *Physical Review*, *77*(1), 1950, p.85.
  - Lumped approximation.

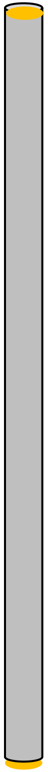
Lumped model

- A lumped, ladder network approximation to the distributed behavior of the real system.
- Similar models are sometimes used for eddy currents; here we model displacement currents as well.
- Why?
  - Accessible to electrical engineers:
    - The model structure can help provide intuition.
    - It can be solved by standard circuit simulators.
  - Natural routes to extend to a nonlinear model.

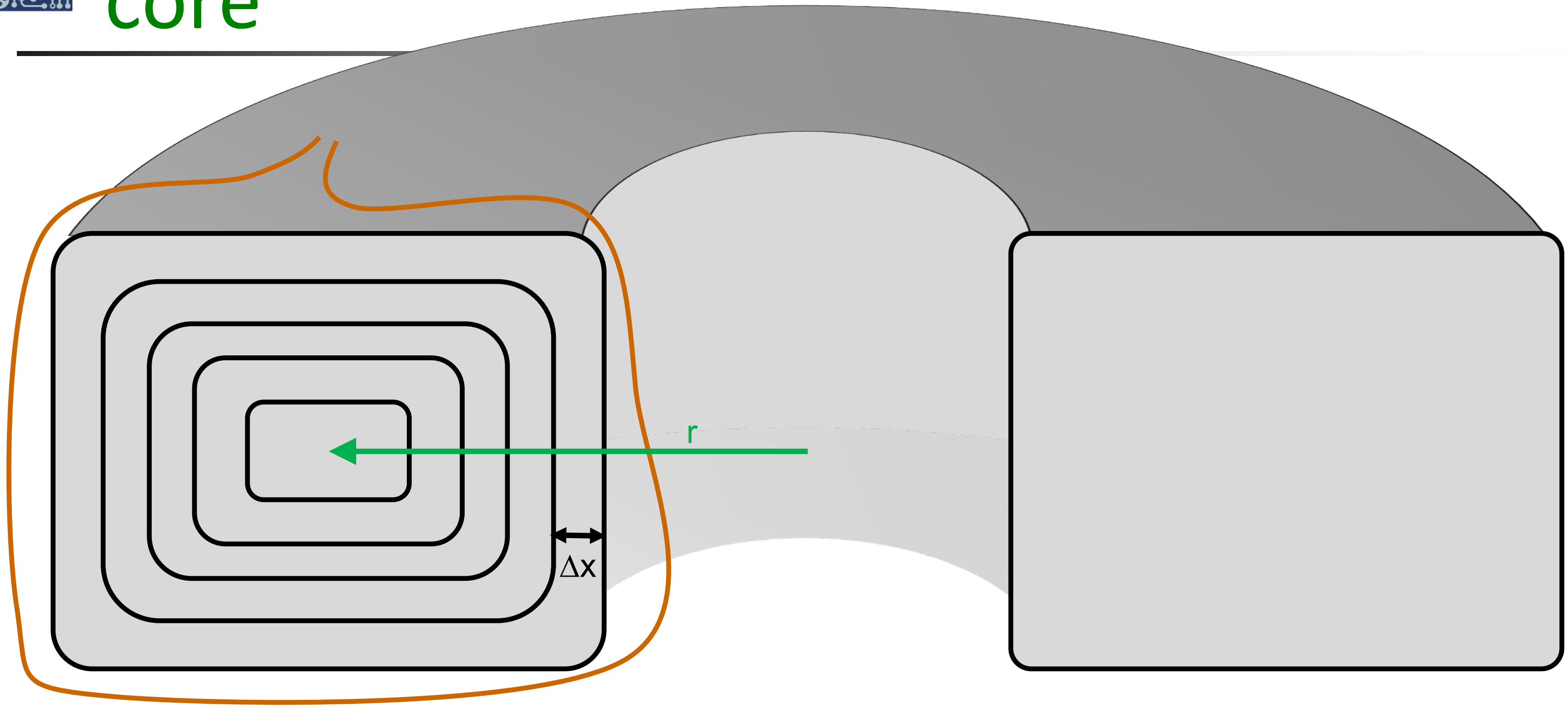
An intuition pump

- ε_r can be on the order of 100,000 (10^5)
- An air capacitor with 50 mm diameter plates, 1 mm apart, has ~ 17.5 pF.

- With $\varepsilon_r = 10^5$, a 5 mm diameter rod, 100 mm long, metalized *at the ends* has 175 pF.

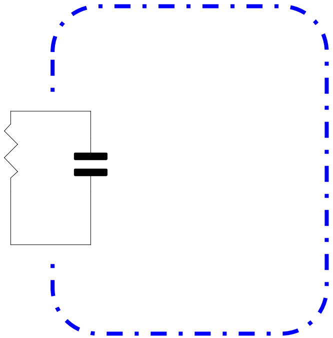


Consider concentric shells of a toroidal core



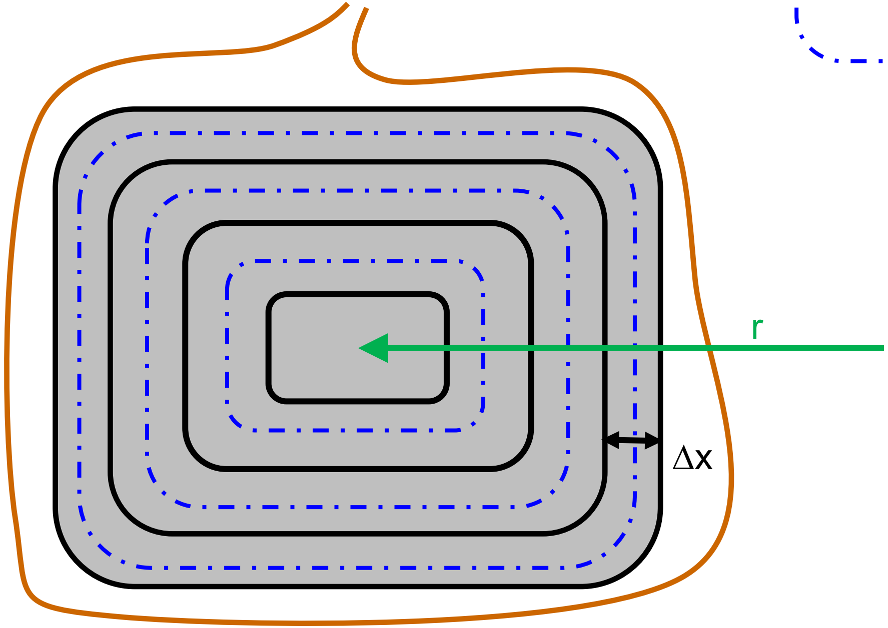
Broken up conceptually, but it's still solid material)

Basis of lumped model



- Each path (blue dashed line) has capacitance and resistance, in parallel:
  - R = $\ell\rho/A$
  - C = $\epsilon_r\epsilon_0 A/\ell$

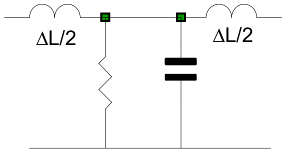
where is the length of that loop, and A is its cross section, $A \approx (2\pi r)\Delta x$.



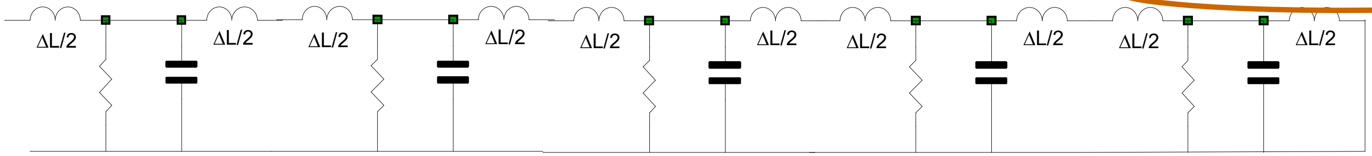
- Each path links a little less flux, as we move from the outer shells to the inner shells.
- The inductance arising from a shell divides half and half: half outside the blue line (not linked) and half inside (linked)
- Inductance of a shell
$\Delta L = N^2 A/(2\pi r) = (\Delta x\, \ell)/(2\pi r)$
(for simplicity assume N =1)

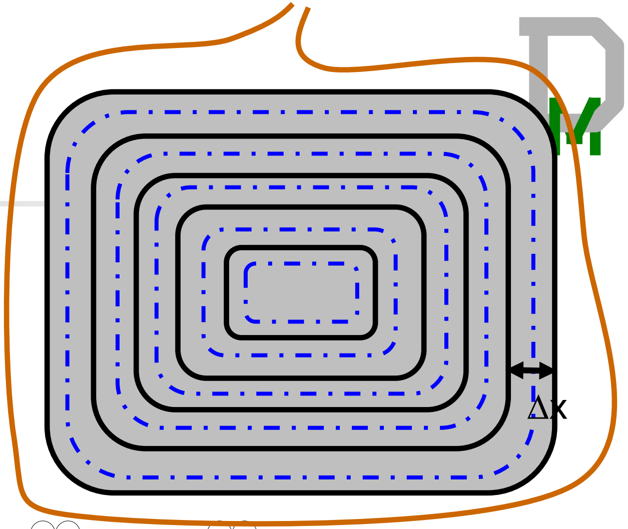
Full lumped model

- Model for one shell



- Transmission line model for multiple shells

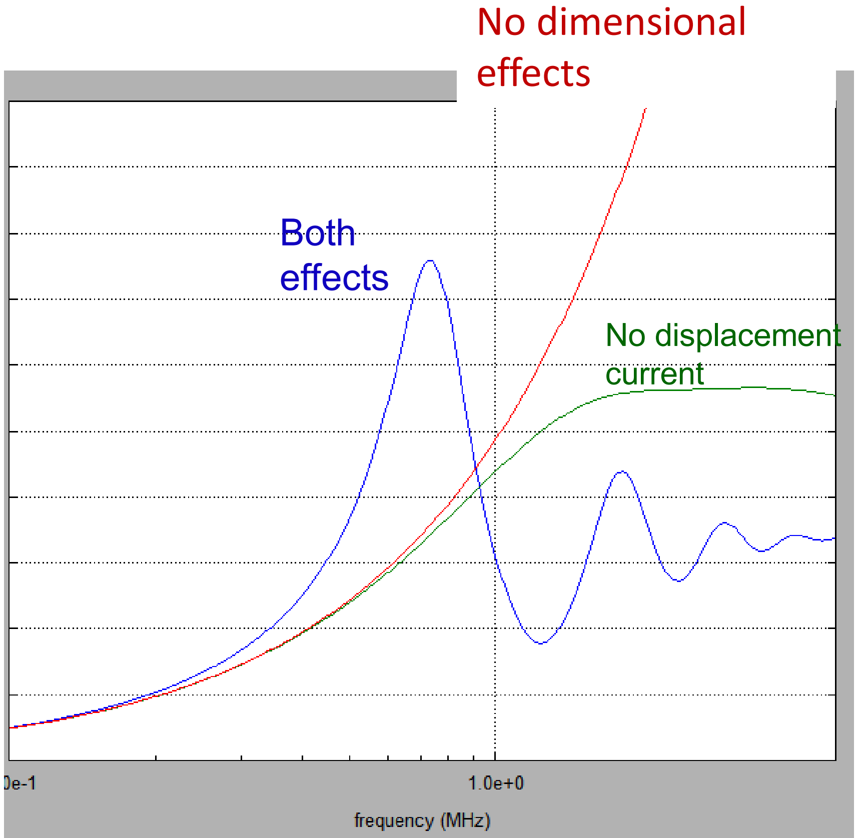




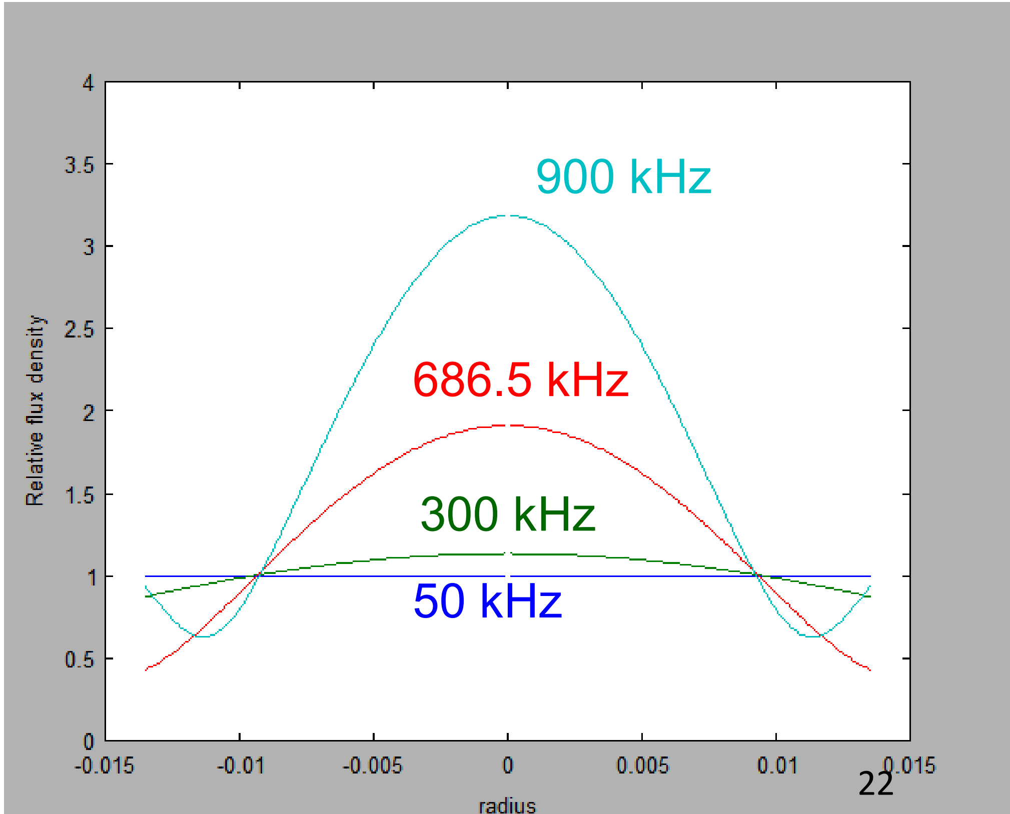
- This is a linear model, but any and all of the components can be made nonlinear and accurately capture nonlinear aspects of the corresponding behavior.
- Add an ideal transformer at the input for $N > 1$.

Lumped model results

Impedance

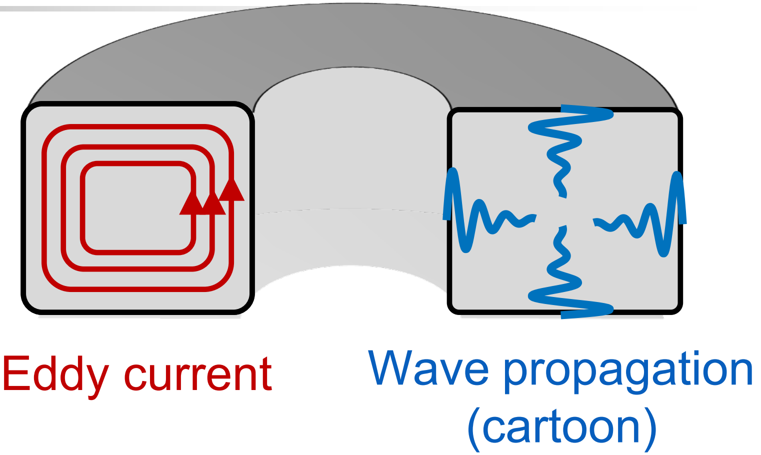


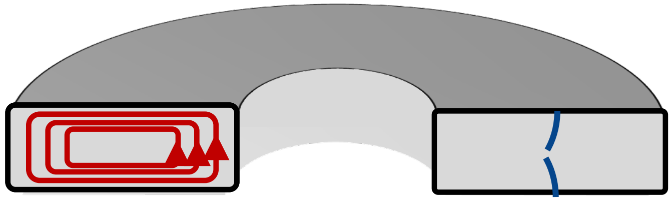
flux density vs. radius



Shape effect on dynamic dimensional effects

- What is the impact of a flat, rectangular cross section?
- Eddy current losses are reduced
  - Just as with steel laminations.
- Wave propagation effects are reduced
  - Distance from surface to interior is reduced.
- For both, the **controlling dimension** is the **small dimension**.



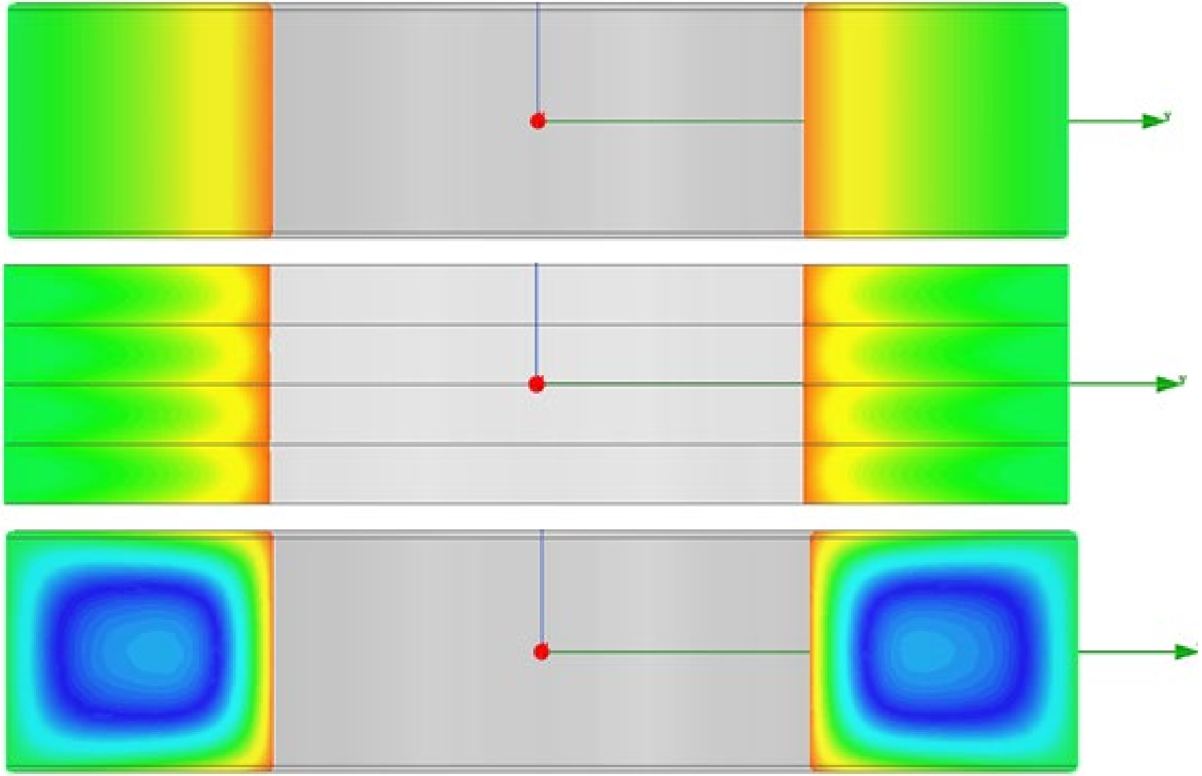


Solution: Stacked Cores

Ideal case

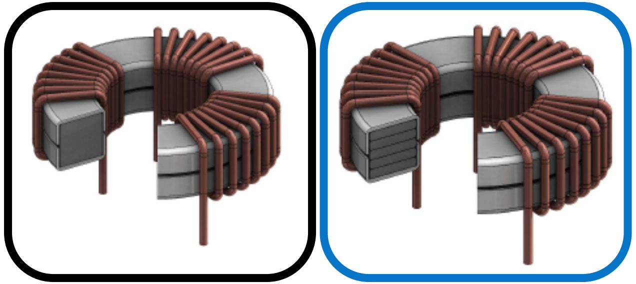
Both effects: stacked

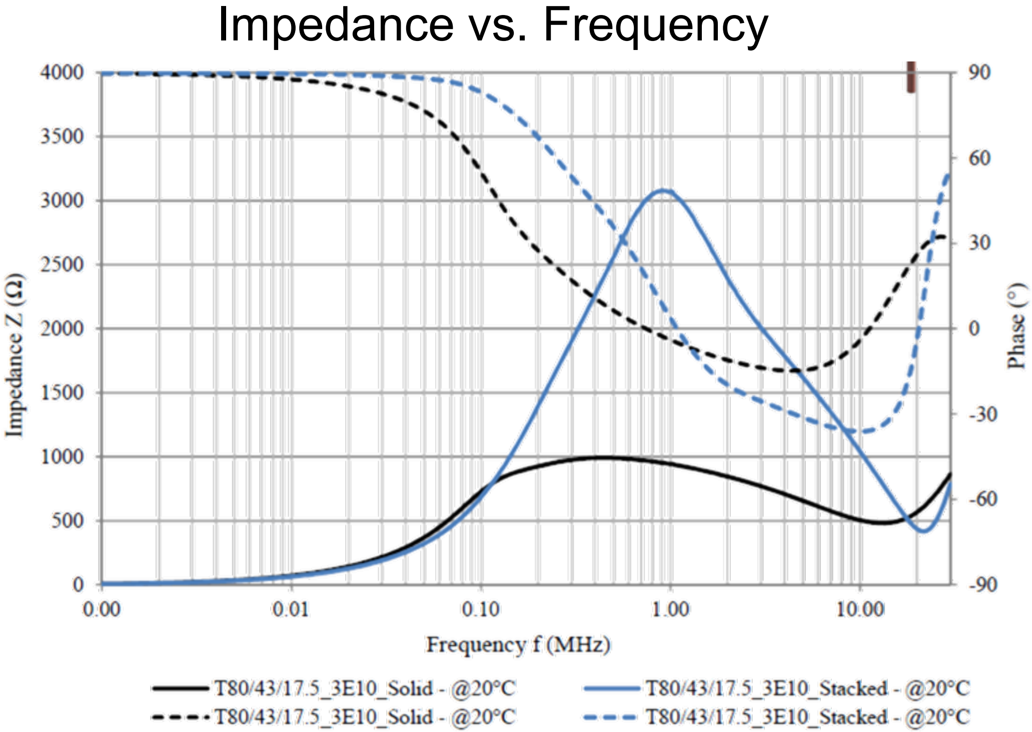
Both effects

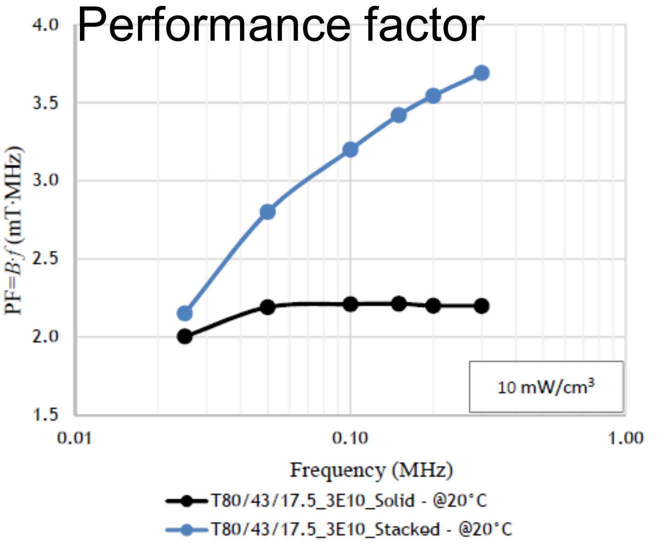


Simulations by Marcin Kacki, SMA Magnetics

Practical test of stacked cores







Effects of Core Shape

- Effects on core loss:
  - Static linear
  - Static nonlinear
  - Dynamic
- **Effects on core fabrication process and material properties.**
- Effects on the field in the winding region:
  - Leakage inductance
  - Winding loss
- Thermal: cooling capability
- Component considerations:
  - Winding fabrication considerations
  - Design parameters: winding length and the winding area/core area ratio.

Shape and core fabrication

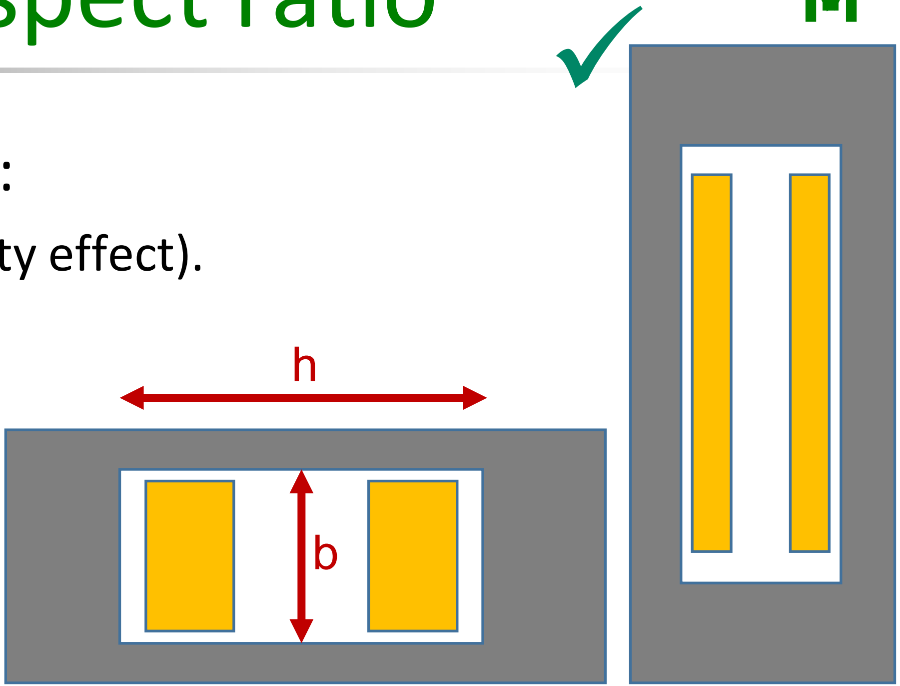
- Process:
  - Press: Want simple shapes, straight sides in one direction.
  - Sintering:
    - Time needed for thicker vs. thinner cores or regions.
    - Shrinkage leads to variations in dimensions.
  - Annealing
  - Machining mating faces.
- Mechanical stability limits shape, size and aspect ratio.
- Effect on material properties   ?

Effects of Core Shape

- Effects on core loss:
  - Static linear
  - Static nonlinear
  - Dynamic
- Effects on core fabrication process and material properties.
- **Effects on the field in the winding region:**
  - Leakage inductance
  - Winding loss
- Thermal: cooling capability
- Component considerations:
  - Winding fabrication considerations
  - Design parameters: winding length and the winding area/core area ratio.

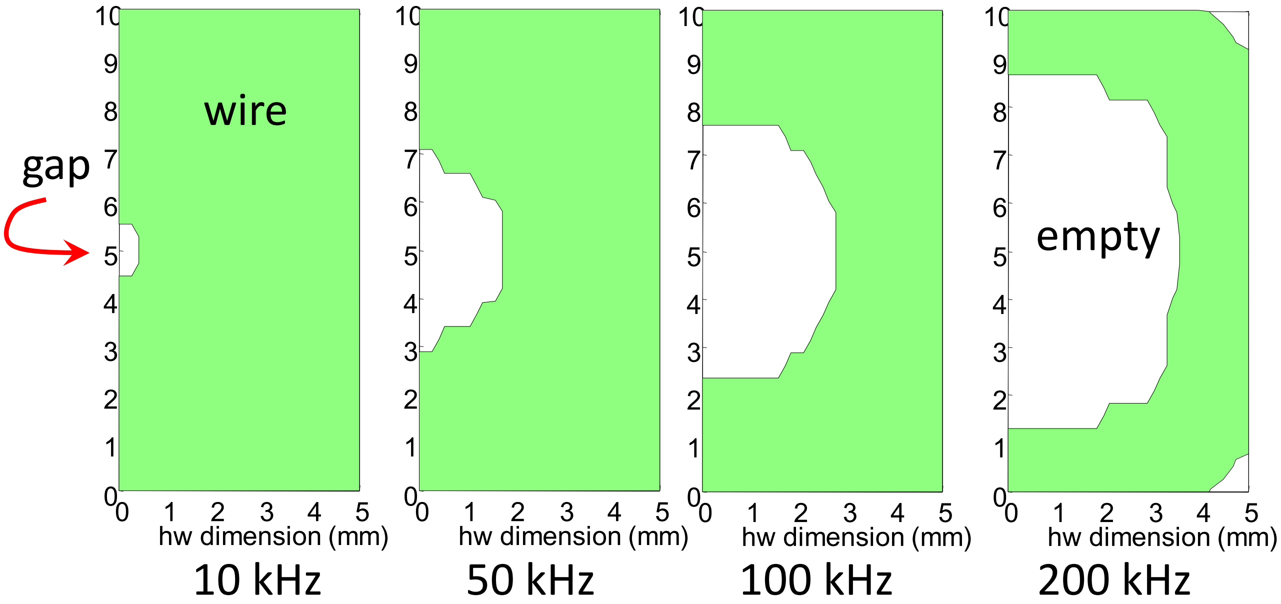
Winding window aspect ratio

- Transformers: large b/h ratio for:
  - Lower ac resistance (lower proximity effect).
  - Lower leakage inductance.
  - Both result from high reluctance in the leakage path.
  - For high isolation voltage, insulation space limits this.
- Inductors:
  - Depends on gap configuration…



Optimum fill factor of an inductor varies

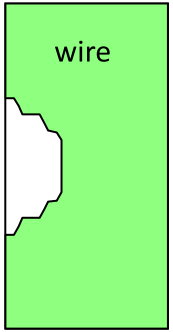
- Trade-off between dc resistance an proximity-effect loss in gap field.
- Depends on ac/dc ratio, frequency and wire strand diameter.
- Does not depend on gap length.



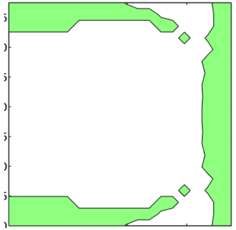
Jiankun Hu, C. R. Sullivan, "Analytical Method for Generalization of Numerically Optimized Inductor Winding Shapes", *IEEE Power Electronics Specialists Conference*, pp. 568–573, June 1999.

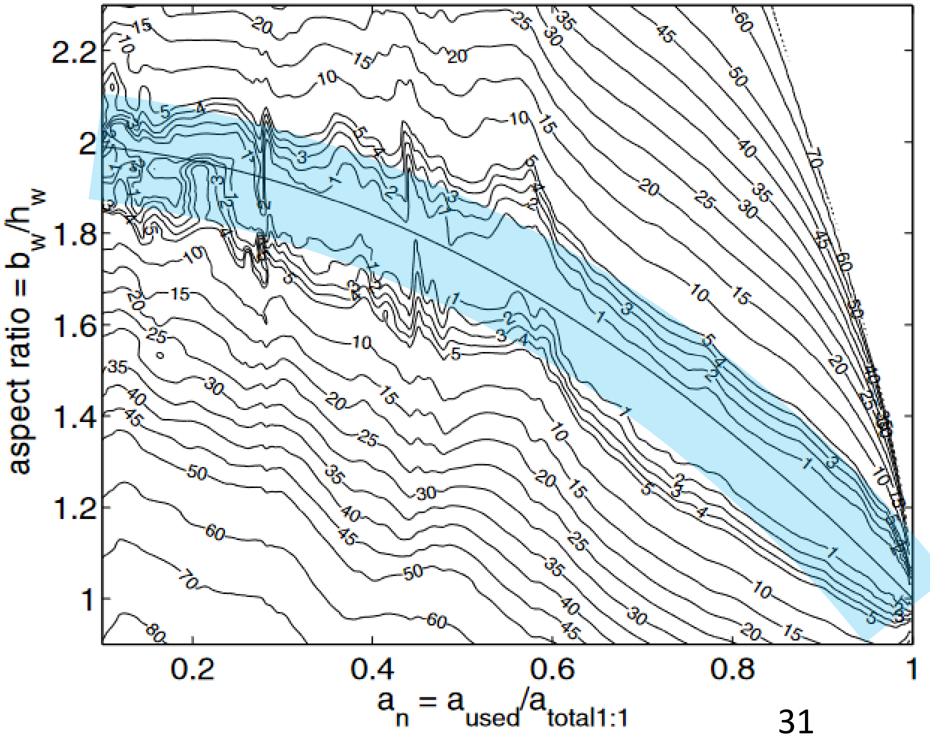
Optimum window aspect ratio depends on fill used for a gapped inductor.

- For a mostly-full window, optimum approaches $b = 2h$



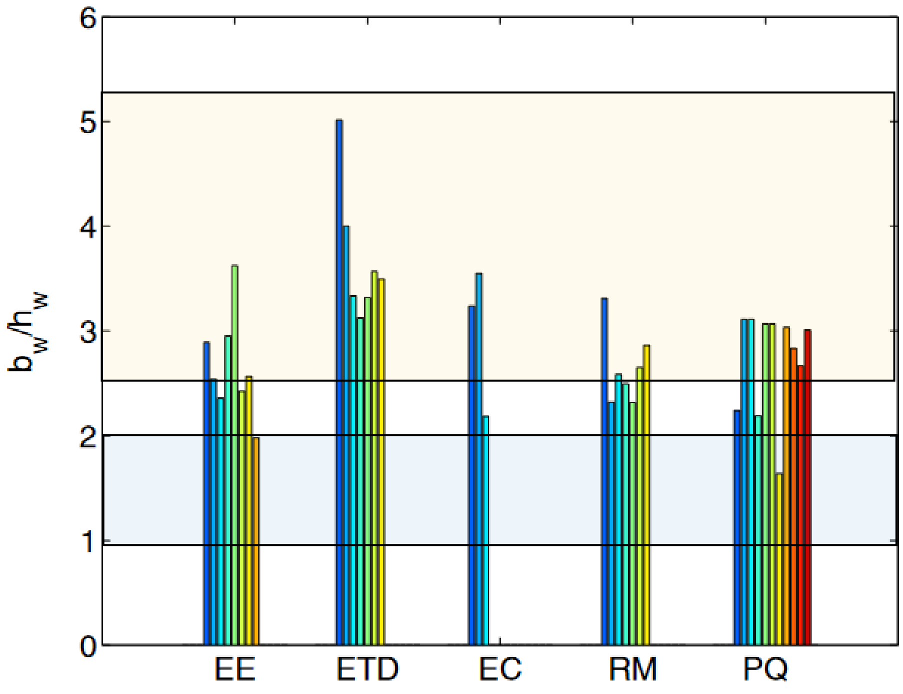
- For a mostly-empty window, optimum approaches $b = h$





R. Jensen, C. R. Sullivan, "Optimal Core Dimensional Ratios for Minimizing Winding Loss in High-Frequency Gapped-Inductor Windings", *IEEE Applied Power Electronics Conference*, pp. 1164–1169, Feb. 2003.

Aspect ratios of common cores



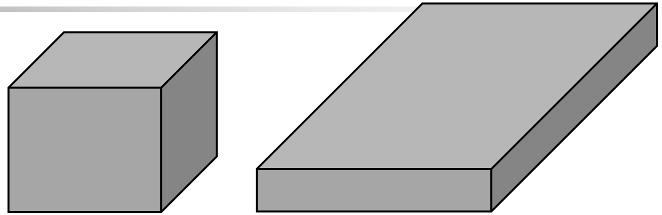
- ETDs and others have a high ratio that's good for transformers.
- One PQ is in the ideal range for inductors; Some RMs and other PQs are close.

Effects of Core Shape

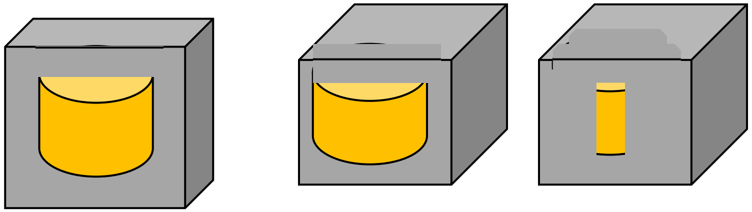
- Effects on core loss:
  - Static linear
  - Static nonlinear
  - Dynamic
- Effects on core fabrication process and material properties.
- Effects on the field in the winding region:
  - Leakage inductance
  - Winding loss

- **Thermal: cooling capability**
- Component considerations:
  - Winding fabrication considerations
  - Design parameters: winding length and the winding area/core area ratio.

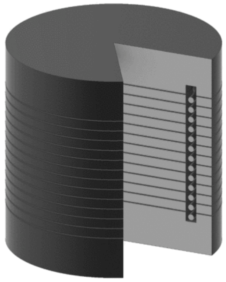
Thermal considerations



- Surface to volume ratio
- Winding and core:
  - Need a plan to get heat out of a winding hidden in a core.



- Floating core pieces:
  - Used in multi-gap designs.
  - Can be hard to get heat out of.
  - Thermally conductive spacers or cores fabricated with integral gaps can solve this problem.



- Large cores: heat conduction from interior to surface.

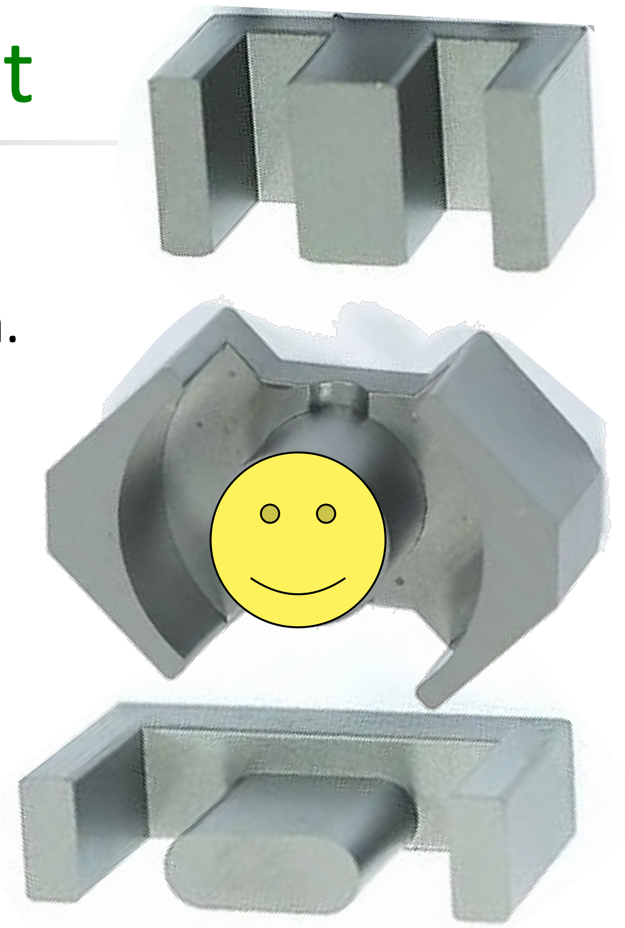
Effects of Core Shape

- Effects on core loss:
  - Static linear
  - Static nonlinear
  - Dynamic
- Effects on core fabrication process and material properties.
- Effects on the field in the winding region:
  - Leakage inductance
  - Winding loss
- Thermal: cooling capability

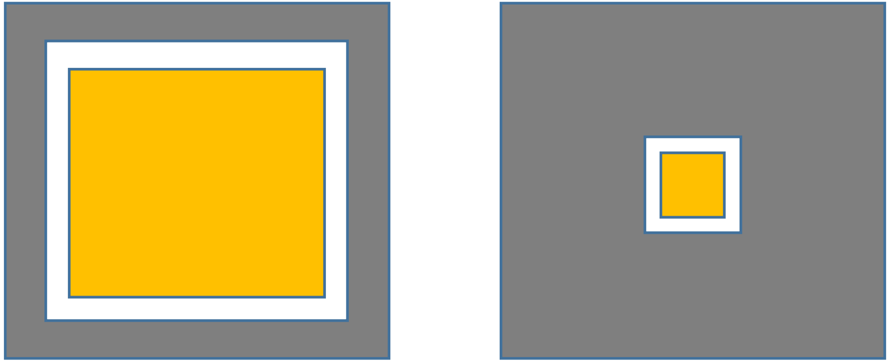
- **Component considerations:**
  - Winding fabrication considerations
  - Design parameters: winding length and the winding area/core area ratio.

Circular center posts are best

- Easiest to wind around (vs. square).
- Lowest winding resistance for a given core area.
- Exceptions:
  - For high power, stacked cores are useful for availability and for avoiding dimensional effects. Rectangles stack better.
  - Oval posts can be useful for low-profile cores: for thermal considerations and for making use of available space.



Winding area vs. core area



- Usually a balance with similar areas is preferred.
  - Too much winding area exacerbates high-frequency winding loss issues.
  - Too much core area exacerbates flux crowding issues.
- Can use to adjust core-loss/winding-loss balance instead of using *N:*
  - If *N* is a small integer, or $N_{opt} < 1$, this is another degree of freedom to use.
- In a saturation-limited design (where core loss is low), using more core area and fewer turns can lead to lower loss.

Summary

- Effects on core loss:
  - Static linear
  - Static nonlinear

  Up to ~15%

  - Dynamic

  Can dominate loss or undermine impedance for large cores at HF
- Effects on core fabrication process and material properties. ??
- Effects on the field in the winding region:
  - Leakage inductance
  - Winding loss

  b >> h for transformers; closer to square for gapped inductors.
- Thermal: cooling capability Want large surface area
- Component considerations:
  - Winding fabrication considerations
  - Design parameters: winding length

   round is good

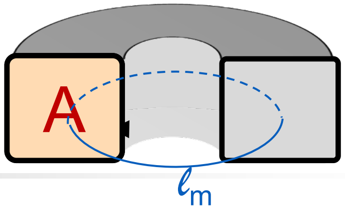
  and the winding area/core area ratio.

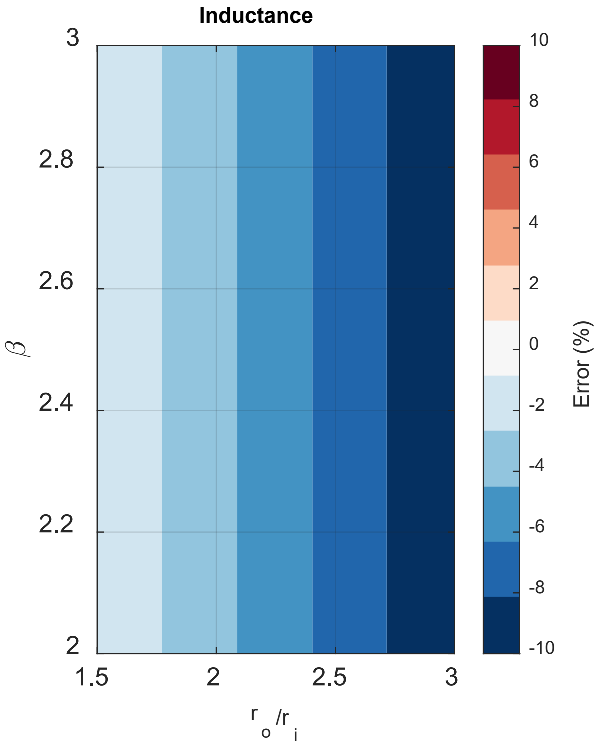
Extra knob to turn, particularly for small N

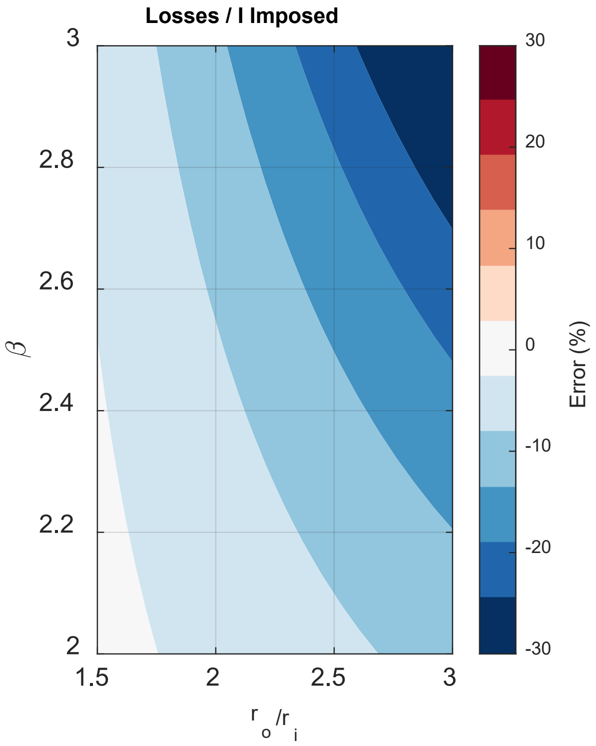
Appendix: analysis of parameters for toroidal cores.

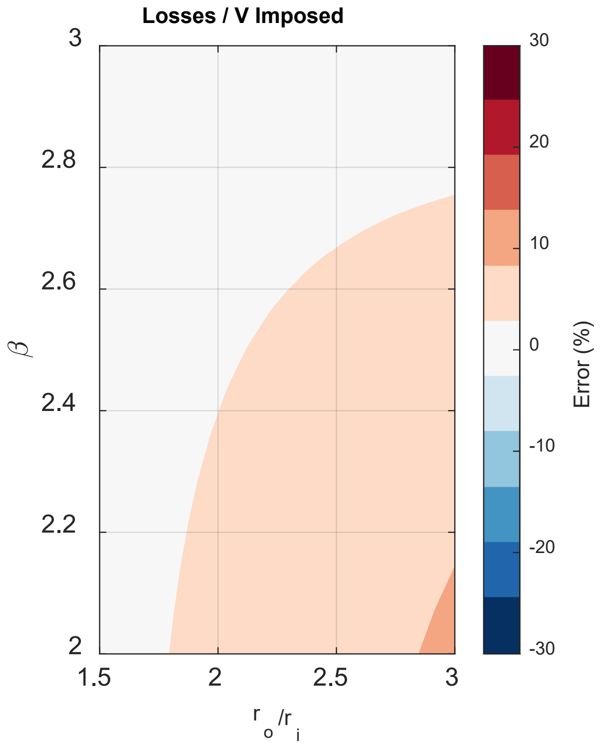
- Linear case:
  - Geometrical parameters
  - Effective parameters
- Nonlinear case

Accuracy with actual area and mean length

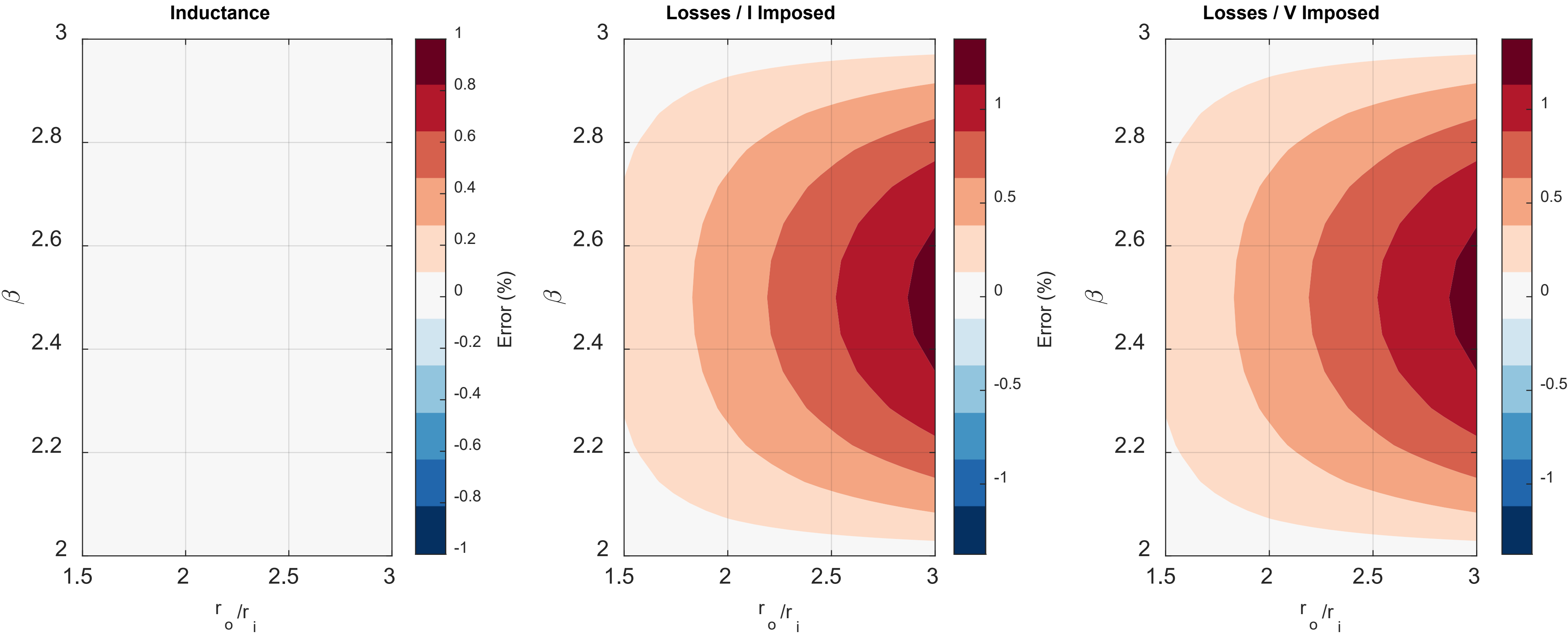




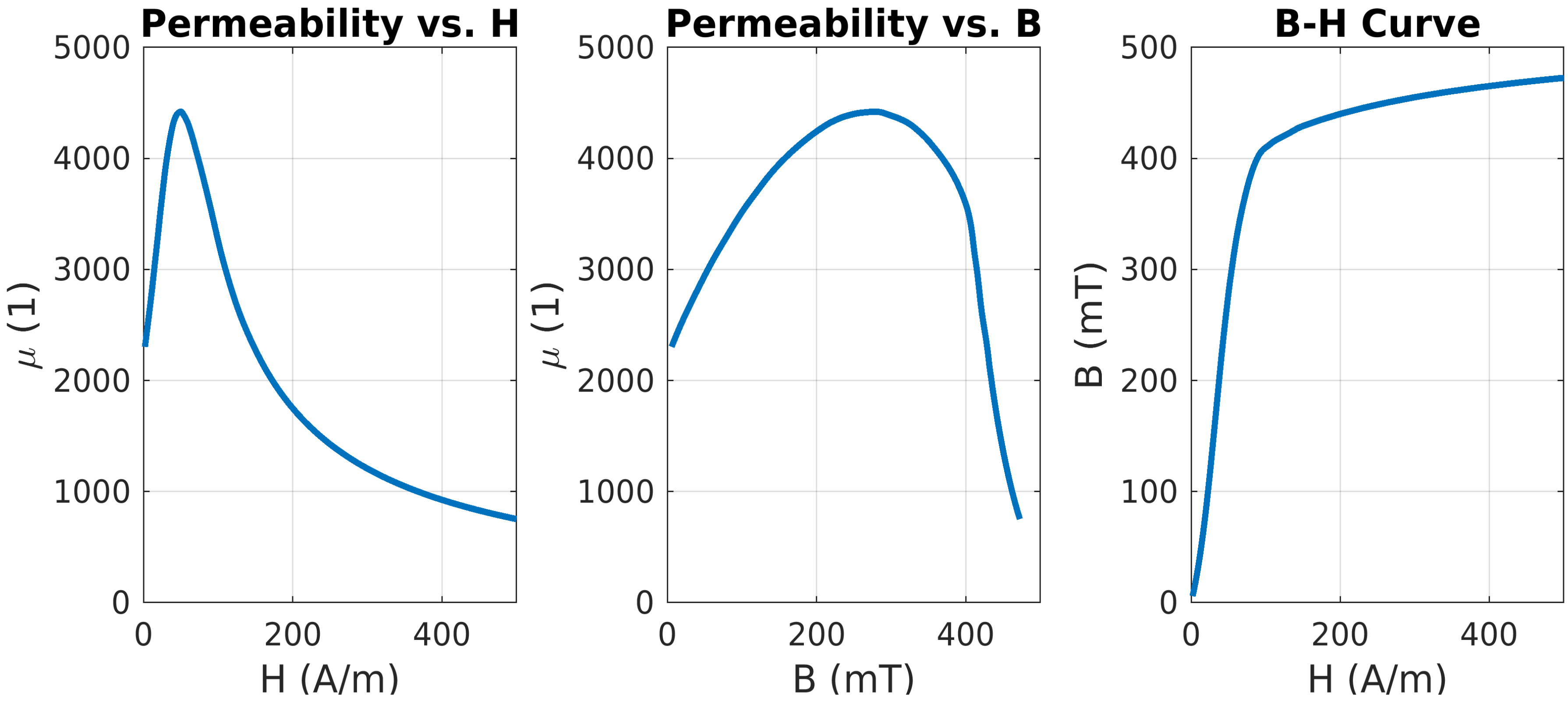




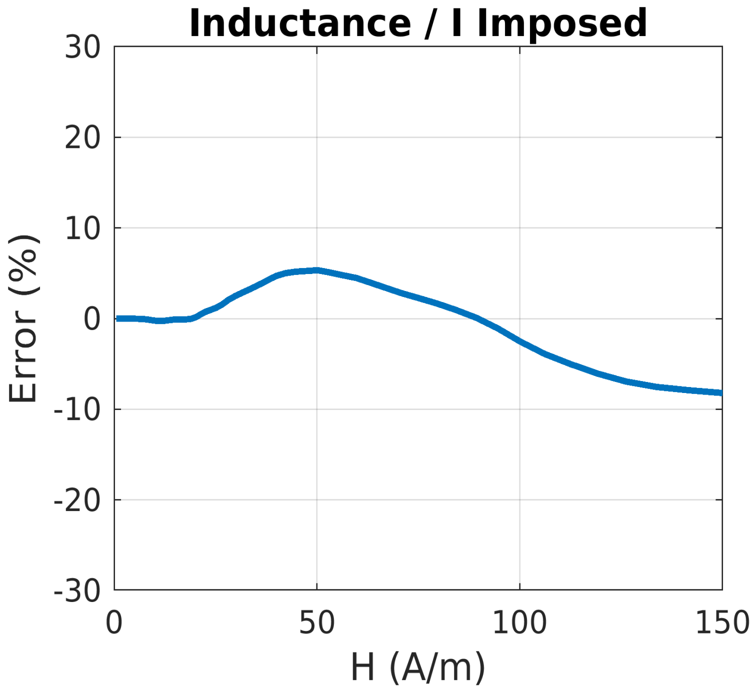
Accuracy with effective parameters (note different scale from previous slide)

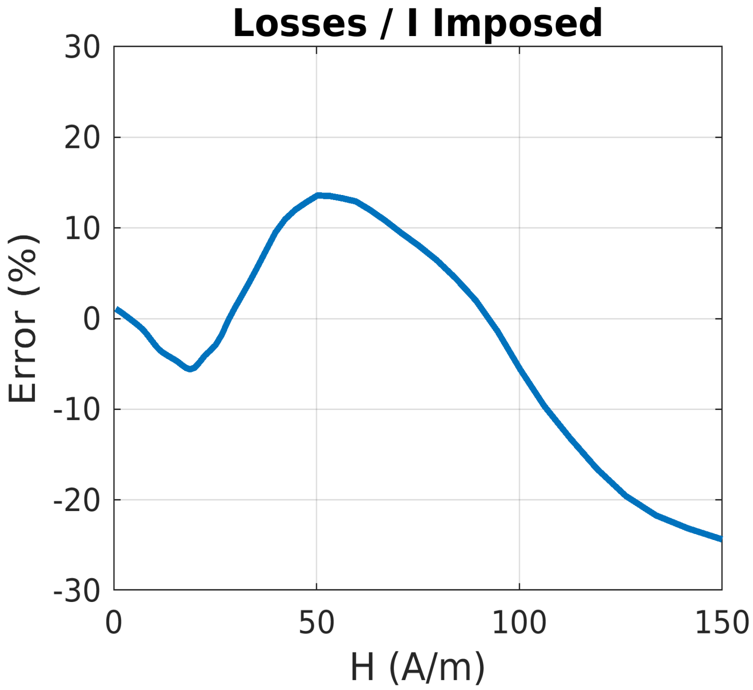


For nonlinear model: N87 Material



Nonlinear model; current fixed, N87 Material





Nonlinear model; voltage fixed, N87 Material