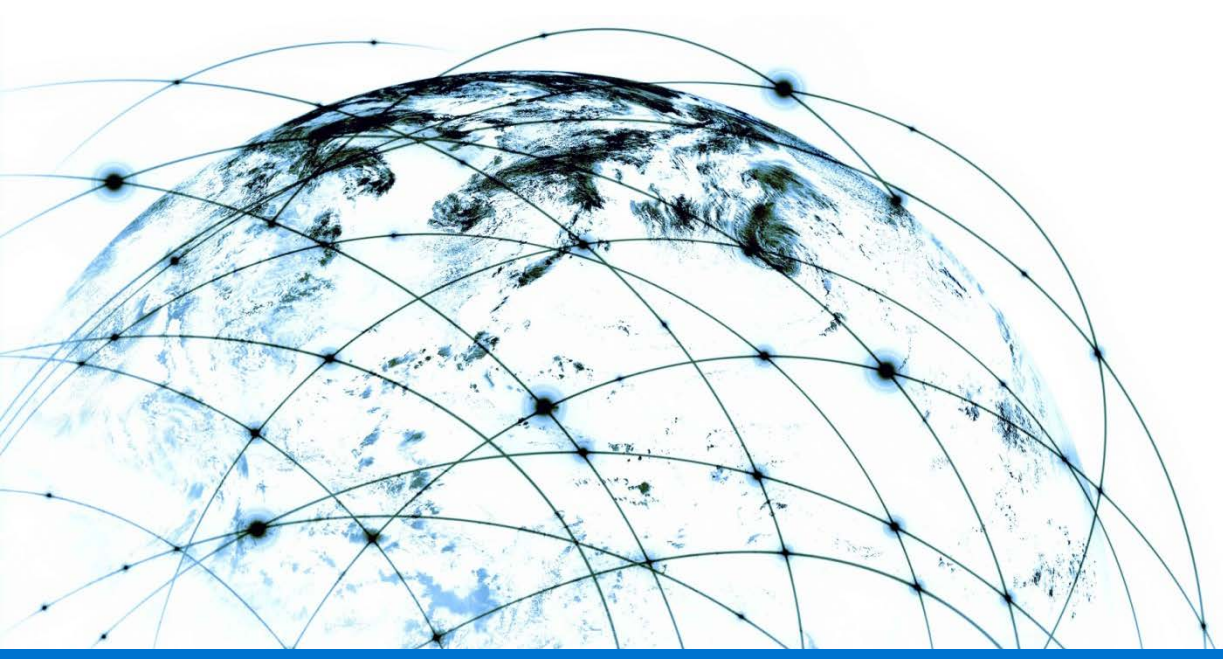

A simple analytical method to calculate air gap induced eddy current losses in inductive components

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Abstract

Simple equations of various loss components in inductive components such as ohmic losses, core losses, proximity losses and skin effect losses are well known and are presented by literature. To calculate air gap induced losses in the winding, time-consuming computer simulation programs with the numerical finite element analysis are commonly used. On the other hand, the existing accurate analytical calculation methods of several authors are mostly not easy to handle for the user and have to be implemented in a computer design tool which doesn´t allow an understanding of what is really going on.

We are proposing a simple analytical method which enables us to estimate the so-called gap losses which are proportional to the proximity losses. The model is based on a simple geometrical consideration which also takes into account the number of distributed air gaps. The derivated equations work in a closed form.

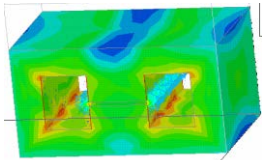
First, the eddy current losses according to Butterworth are explained. Then the general stray field influence is modeled by using the fringing factor as a stray field cross section equivalent. Afterwards, the model is expanded to a location-dependent stray field which considers the influence of the distance of the winding to the air gap. The evaluation of the equations is done with the finite element analysis by using several examples. The comparison shows a satisfying correlation. In the end, the algorithm is simplified to a rule of thumb which enables first approximation of gap losses by hand. The rule can be used for a quick and simple double-check of the inductor layout, for double-checking computer calculation, or it can be used as a design rule.

In today‘s presentation

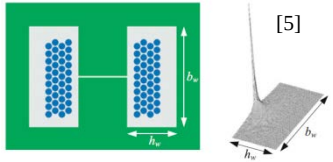
1. Current Situation
2. Fringing factor
3. Eddy current losses according to Butterworth
4. Model for gap losses
5. Determine gap losses with the help of proximity losses
6. Simplification
7. Evaluation via simulation
8. Limits and weakness
9. Rule of thump

Current Situation

- To calculate air gap induced losses in the winding, time-consuming numerical computer simulation programs with the numerical finite element analysis are commonly used [1]



- On the other hand existing accurate analytical calculation methods are not easy to handle for the user or have to be implemented in a computer design tool [2, 3]



[5]

- Simple equations of various loss components in inductive components such as Ohmic losses, core losses, proximity losses and skin effect losses are well known [4]

$$D(x) = 2x \cdot \frac{\sinh(x) - \sin(x)}{\cosh(x) + \cos(x)}$$

- A simple analytical equation which enables first approximation of gap losses by hand is to our knowledge still missing

$$\frac{P_{gap}}{P_{prox}} \approx \frac{3}{4} \frac{b_w}{h_w n}$$

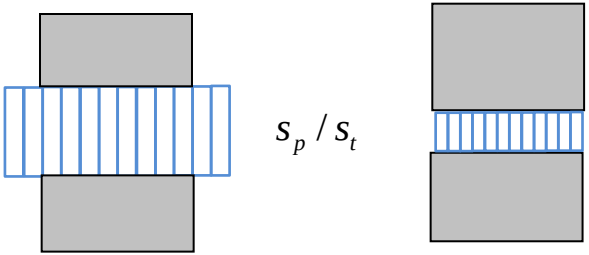
1. http://www.femm.info/wiki/HomePage
2. Albach M, Roßmanith H (2001 The influence of air gap size and winding position on the proximity losses in high frequency transformers. PESC, Vancouver, Canada, S 1485-1490
3. Jensen RA, Sullivan CR (2003) Optimal core dimensional ratios for minimizing winding loss in high-frequency gapped-inductor windings. APEC, Bd 2, S 1164–1169
4. Dowell PL (1966) Effects of eddy currents in transformer windings. PROC IEE, Bd 113, Nr 8, S 1387-1394
5. Albach M (2017) Induktivitäten in der Leistungselektronik. Springer Vieweg

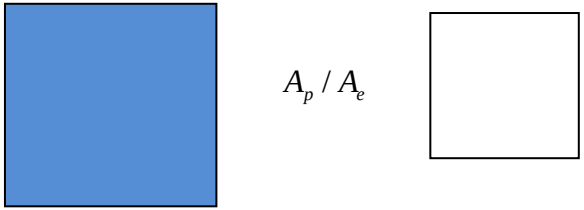
Definition of the fringing factor

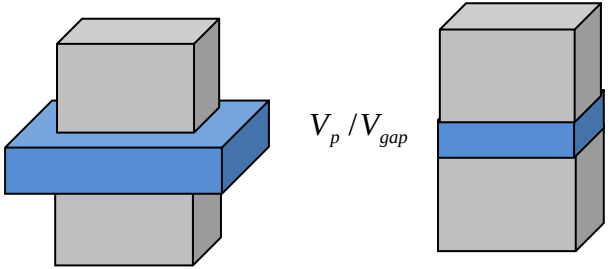
The fringing factor F describes the increase of the effective magnetic cross section in the area of the air gap [1]:

$$F = \frac{s_p}{s_t} = \frac{A_p}{A_e} = \frac{V_p}{V_{gap}} \qquad with: \quad s_t \approx \frac{\mu_0 \cdot A_e}{A_L}$$

A_e: effective cross section area
A_L: normalized inductivity
V_p: fringing volume
V_{gap}: volume of air gap







We have the integral information about the stray field volume V_w:

$$V_w = (A_p - A_e)s_p$$

$$V_w = V_{gap}(F - 1)$$

V_w

The fringing volume V_w is the key for the following model

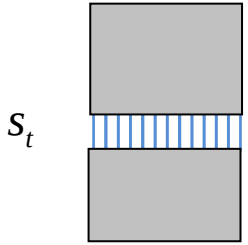
1. Snelling EC (1988) Soft Ferrites – Properties and Applications.. Butterworths, second edition

Calculation of the fringing factor

Following is a simple equation based on $ds=s_p/2$ from Mohan [1]:

$$F=\frac{(b_k+s_p/n)(h_k+s_p/n)}{b_k h_k}\approx 1+2{,}5\frac{s_p/n}{\sqrt{A}}$$

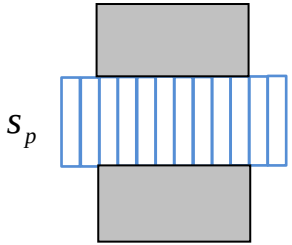
b_k: with of center leg
h_k: depth of center leg



McLyman gives the most popular correlation with s_p as air gap and A_e as magnetic cross section [2]:

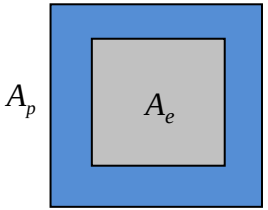
$$F=1+\frac{s_p/n}{\sqrt{A_e}}\ln\frac{2b_w}{s_p/n}\approx\frac{1}{1-1{,}2\frac{s_t^{0,7}b_w^{0,3}}{n^{0,7}\sqrt{A_e}}}\approx\frac{1}{1-2{,}5\frac{s_t}{n\sqrt{A}}}$$

b_w: length of winding space
n: number of distributed air gaps



Expanding to rectangular cross section with U_k as circumference of A_e [3] :

$$F=1+\frac{s_p/nU_k}{2A_e}\ln\left(\frac{2b_w}{s_p/n}\right)\approx\left(1-0{,}3\frac{s_t^{0,7}b_w^{0,3}U_k}{n^{0,7}A_e}\right)^{-1}$$



1. N. Mohan, T. M. Undeland, and W. P. Robbins -"Power Electronics –Converter, Applications, and Design", John Wiley & Sons, Inc., 2003
2. McLyman WT (2004) Transformer and Inductor Design Handbook. Marcel Dekker, New York
3. Albach M (2017) Induktivitäten in der Leistungselektronik. Springer Vieweg

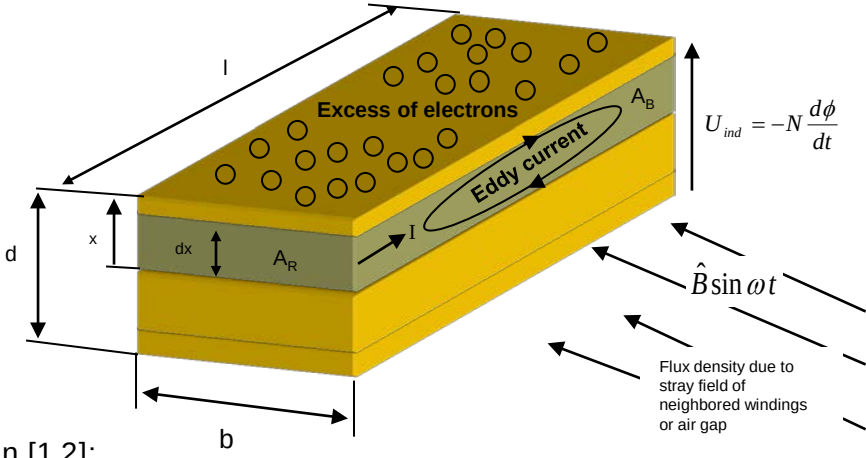
Eddy current losses according to Butterworth

For a sinus similar stray field a voltage U_{eff} is induced [1,2]:

$$U_{ind} = -N\frac{d\phi}{dt} \quad \rightarrow \quad U_{eff} = \frac{\omega N \hat{B} A_B}{\sqrt{2}} = \frac{\omega \hat{B} 2xl}{\sqrt{2}}$$

The electron compensation due to the conductivity in longitudinal direction results in power losses [1,2] :

$$dP_{prox} = \frac{U_{eff}^2}{R} = \frac{\omega^2 \hat{B}^2 lbx^2}{\rho_{cu}} dx \qquad with: \; R = \frac{2\rho_{cu} l}{bdx}$$



By integration dx we get the power losses [5,6]:

$$P_{prox} = \frac{\omega^2 \hat{B}^2 lb}{\rho_{cu}} \int_0^{d/2} x^2 dx = \frac{\omega^2 \hat{B}^2 lbd^3}{24\rho_{cu}}$$

The well-known equation for a round wire results with $b=3\pi/16 \cdot d$.
The flux density B^2 is perpendicular to the axis in longitudinal direction [1,2]:

$$P_{prox} = \frac{\pi\omega^2 \hat{B}^2 ld^4}{128\rho_{cu}}$$

1. Snelling EC (1988) Soft Ferrites – Properties and Applications.. Butterworths, second edition
2. Butterworths, s.: 'Eddy-current losses in cylindrical conductors with special application to the alternating current resistance of short coils', Phil. Trans., [A], 1921, 222, p. 57

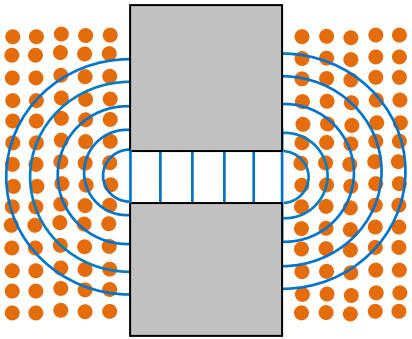
Proximity losses proportional to the winding volume

Substitution of the total length l of wire in the proximity losses with the help of the filling factor $p_G = A_{cu}/A_F$:

$$P_{prox} = \frac{\pi \omega^2 \hat{B}^2 l d^4}{128 \rho_{cu}} \qquad with: \; l = \frac{4 \cdot p_G \cdot A_F \cdot l_w}{d^2 \pi}$$

For a round wire diameter d_0 we get the proximity losses specified as a function of $B^2 A_F$:

$$P_{prox} = \frac{\pi^2}{8} \frac{p_g}{\rho_{cu}} \cdot l_w \cdot d_0^{\,2} \cdot f^2 \cdot \hat{B}^2 \cdot A_F$$



Induced losses in copper wire by the fringing field

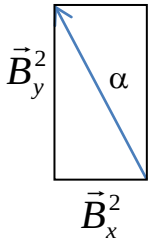
Adapted to the volume of winding space V_F we get the losses proportional to $B^2 V_F$. This equation is only valid for a small resistance factor F_r<2 :

$$P_{prox} = \frac{\pi^2}{8} \frac{p_g}{\rho_{cu}} \cdot d_0^{\,2} \cdot f^2 \cdot \hat{B}^2 \cdot V_F$$

Additional proximity loses due to the air gap

Additional proximity losses results from fringing flux density $B(x,y)_{gap}$ of the air gap:

$$P_{prox} = \frac{\pi^2}{8}\frac{p_g}{\rho_{cu}} d_0{}^2 f^2 \cdot \left(\vec{B}(x,y)_{wire} + \vec{B}(x,y)_{gap}\right)^2 V_F$$

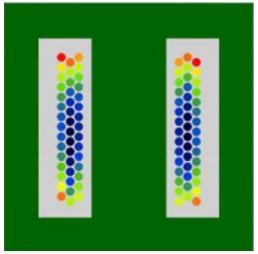


After solving the bracket we get an expression containing the scalar product:

$$P_{prox} = \frac{\pi^2}{8}\frac{p_g}{\rho_{cu}} d_0{}^2 f^2 \cdot \left(\hat{B}_{wire}^2(x,y) + \hat{B}_{gap}^2(x,y) + 2\hat{B}_{wire}(x,y)\ \hat{B}_{gap}(x,y)\cos\alpha\right) \cdot V_F$$

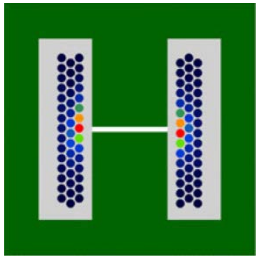
Location of the wire field and gap field:

a) The classical proximity losses are high in outer winding space due to missing field compensation due to missing proximity of a wire



distribution of proximity losses P_{prox}=1,3mW; constant current [1]

b) The additional losses due to the air gap are high in the center of the component



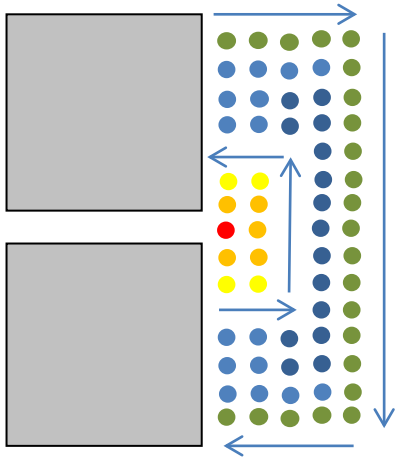
distribution of proximity losses P_{prox}=9,5mW including a air gap [1]

1. Albach M (2017) Induktivitäten in der Leistungselektronik. Springer Vieweg

Definition of the gap losses

As a result classical proximity losses not appear in the same position as the additional proximity losses due to air gap. This results that the scalar product can be neglected.

$$P_{prox} \approx \frac{\pi^2}{8}\frac{p_g}{\rho_{cu}} l_w d_0^{\,2} f^2 \cdot \left(\hat{B}_{wire}^2 + \hat{B}_{gap}^2\right) \cdot V_F \qquad with: \left|\vec{\hat{B}}(x,y)_{wire} - \vec{\hat{B}}(x,y)_{gap}\right| >> 0$$



Location and direction of the wire field and gap field

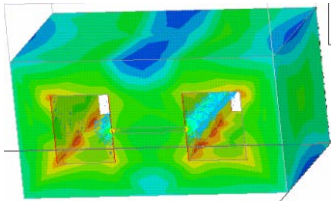
We get two field components which are independent from each other.
Therefore the gap losses can be defined in first approximation as a separate loss component:

$$P_{gap} \approx \frac{\pi^2}{8}\frac{p_g}{\rho_{cu}} l_w d_0^{\,2} f^2 \left(\hat{B}_{gap}^2 V_F\right)$$

Model for the gap losses

In order to get simple equations, the basic idea is to solve in a bulk and not wire by wire.
Now we take into account that the local B is not constant within the winding volume V.

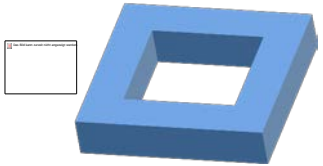
$$P_{gap} = \frac{\pi^2}{8}\frac{p_g}{\rho_{cu}} \cdot d_0^{\,2} \cdot f^2 \cdot \hat{B}(V)^2 V$$



Fringing flux in the winding volume

Generating a type of function for *B(V)* with the requirement B(V)V is constant according to a constant magnetic moment from a magnetic dipole [1].

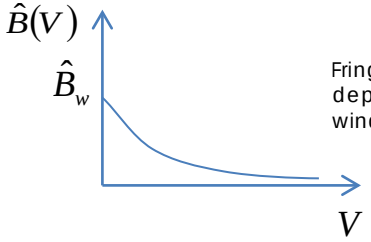
$$\hat{B}(V) \cdot V = const \qquad \hat{B}(V)V = \hat{B}_w V_w \qquad with: \; B_w = \frac{B_{core} A_e}{A_p} = \frac{B_{core}}{F}$$



Fringing flux volume equivalent

Now we insert the initial condition *B(0)=B$_w$ and we get:*

$$\hat{B}(V) = \frac{\hat{B}_w V_w}{V_w + V}$$



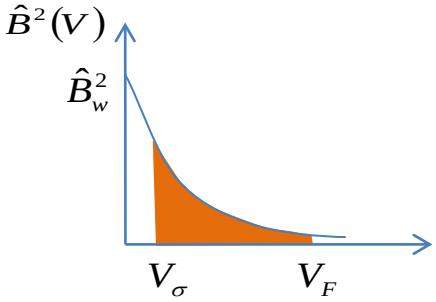
Fringing flux in dependence of winding volume

1. Kohlrausch (1996) Praktische Physik, Band2. B.G. Teubner Stuttgart. 24. Auflage

Model for the gap losses

Only regarding the volume of windings which are lying in the field *B(V) we get via integration the product B²(V)V*:

$$\hat{B}^2(V)\cdot V = \int_{V_\sigma}^{V_F} \hat{B}^2(V)dV = \int_{V_\sigma}^{V_F}\left(\frac{\hat{B}_w V_w}{V_w+V}\right)^2 dV = \hat{B}_w^2\cdot V_w\left(\frac{V_w}{V_w+V_\sigma}-\frac{V_w}{V_w+V_F}\right)$$

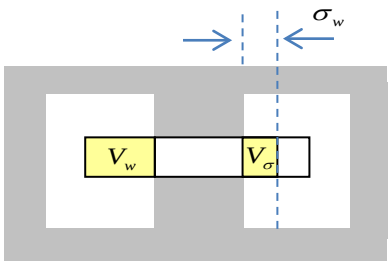


We get a winding trans flux number *0<k<1*:

$$k = \left(\frac{V_w}{V_w+V_\sigma}-\frac{V_w}{V_w+V_F}\right) \approx \frac{1}{1+V_\sigma/V_w}$$

$$with:\ V_\sigma \approx \sigma_w s_p U_k$$
$$V_w = V_{gap}(F-1)$$

V_σ: volume of air gap upholstery
V_F: effective winding space
σ_w: distance to the air gap
U_k: circumference of A_e



Now we get a simple equation for the gap losses including the volume of the air gap V_{gap}:

$$P_{gap} = \frac{\pi^2}{8}\frac{p_g}{\rho_{cu}}\cdot d_0^{\,2} f^2 \cdot \hat{B}_{core}^{\,2} V_{gap}\frac{(F-1)}{F^2}\cdot k$$

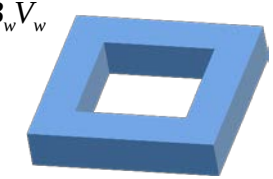
$$with:\quad B_w = \frac{B_{core}}{F}$$

(only valid for low resistance factor F_r<2)

Determine gap losses with the help of proximity losses

The gap losses P_{gap} can be determined with the proximity losses P_{prox} by dividing both equations.
This rescaling is useful because the accurarity of the P_{gap} improves according to the accuratiry of the P_{prox}.

$$\frac{P_{gap}}{P_{prox}} = \frac{\hat{B}_{gap}^2 V_F}{\hat{B}_{wire}^2 V_F} = \frac{\hat{B}_w^2 V_w}{\hat{B}_{wire}^2 V_F} k \qquad \hat{B}_{gap}^2 = \frac{\hat{B}_{core}^2 V_{gap}}{V_F} \frac{(F-1)}{F^2} k$$



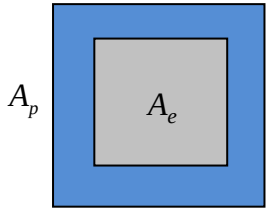
fringing flux volume equivalent

The middle flux density in the winding space is according [1] : $\overline{B}^2 = \frac{1}{3}\left(\frac{\mu_0 N \hat{I}}{\sqrt{2} b_w}\right)^2$ $\quad with: \ \hat{B}^2 = 2\overline{B}^2$
As a result we get the B-Field in the winding space to:

$$\hat{B}_{wire}^2 = \frac{1}{3}\left(\frac{\mu_0 \hat{B}_{core} A_e}{b_w A_L}\right)^2 \qquad with: \ \hat{I} = \frac{\hat{B}_{core} A_e}{N A_L};$$

The fringing flux density B_w is the corresponding flux density equal to the cross section of fringing air gap A_p:

$$\hat{B}_w = \frac{\hat{B}_{core} A_e}{A_p}$$



the increase of effective cross section due to fringing

Simplification via setting both B-equations into the rescaled relation above:

$$\frac{P_{gap}}{P_{prox}} = \frac{3 V_w A_L^2 b_w^2}{\mu_0^2 A_p^2 V_F} k$$

1. Snelling EC (1988) Soft Ferrites – Properties and Applications.. Butterworths, second edition

Simplification with substitution of some parameters

Regarding the volume ratio we get: $$\frac{P_{gap}}{P_{prox}} = \frac{3A_L^2 b_w^2 V_w}{\mu_0^2 A_p^2 V_F} k$$

Fringing field volume V_w can be described by the effective air gap area A_p and the effective air gap s_p : $V_w = (A_p - A_e) s_p$

The effective air gap area is proportional to the fringing factor: $A_p = F A_e$

The effective air gap s_p can be described by fringing: $s_p = F s_t$

The air gap s_t without fringing is: $s_t = \mu_0 \cdot A_e / A_L$

The winding trans flux number: $$k = \left(1 + \frac{V_\sigma}{V_w}\right)^{-1} = \left(1 + \frac{\sigma_w \cdot U_k}{A_e (F-1)}\right)^{-1}$$

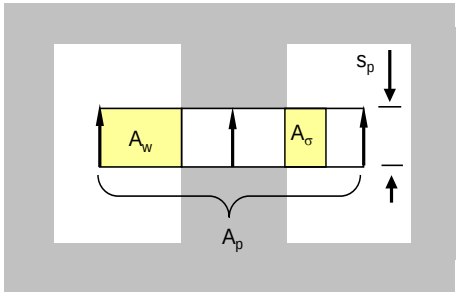
This results in: $$\frac{P_{gap}}{P_{prox}} = \frac{3A_L b_w^2}{\mu_0 V_F}\left(\frac{F-1}{F}\right)k \quad or \quad \frac{P_{gap}}{P_{prox}} = \frac{3b_w^2}{V_F}\left(\frac{A_L}{\mu_0} - \frac{A_e}{s_p}\right)k$$

With modificated fringing from McLyman we get the simplification: $$F = \left(1 - 2{,}5\frac{s_t}{n\sqrt{A}}\right)^{-1}$$

$$\frac{P_{gap}}{P_{prox}} \approx \frac{7{,}5 b_w \sqrt{A_e}}{n h_w l_w} k$$

Simplification based on the area ratio via Mohan

Regarding the area relation we get: $\frac{P_{gap}}{P_{prox}} = \frac{3A_L^2 b_w^2 V_w}{\mu_0^2 A_p^2 V_F} k \rightarrow \frac{P_{gap}}{P_{prox}} \frac{3A_w A_L^2 b_w^2}{\mu_0^2 A_p^2 A_F} k$



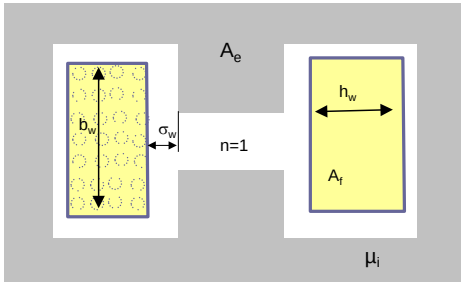
The stray field window A_w with the extension ds and the length of gap s_p is: $A_w = ds \cdot s_p$

The stray field extension ds is according [1]: $ds = \frac{s_p}{2}$

Including the number of distributed air gap n we get : $ds = \frac{s_p}{2n}$

The actual cross section of fringing air gap A_p: $A_p = FA_e$

The actual total air gap s_p can be describe by the fringing factor: $s_p = Fs_t$



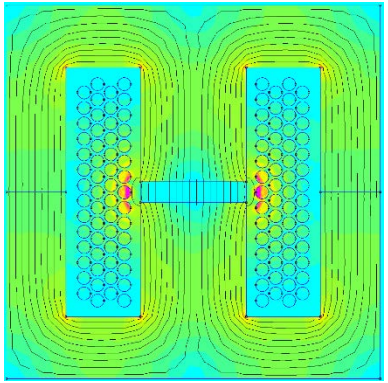
The air gap s_t without fringing is: $s_t = \frac{\mu_0 \cdot A_e}{A_L}$

This results in: $\frac{P_{gap}}{P_{prox}} \approx \frac{3b_w}{2h_w n} k$

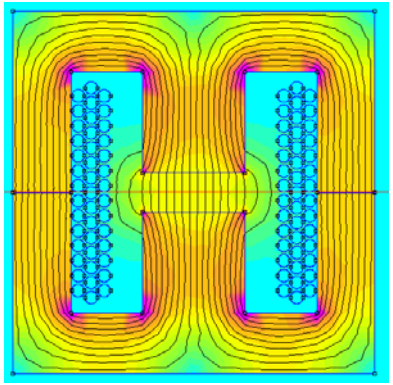
$$with: \; k = \frac{1}{1 + \frac{A_\sigma}{A_w}} = \frac{1}{1 + \frac{2n\sigma}{s_p}}$$

1. N. Mohan, T. M. Undeland, and W. P. Robbins -"Power Electronics –Converter, Applications, and Design", John Wiley & Sons, Inc., 2003

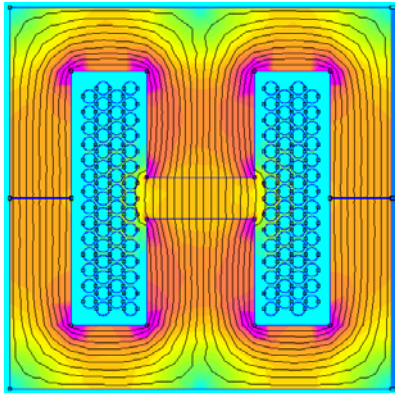
Simulation tools for evaluation at different conditions [1;2]



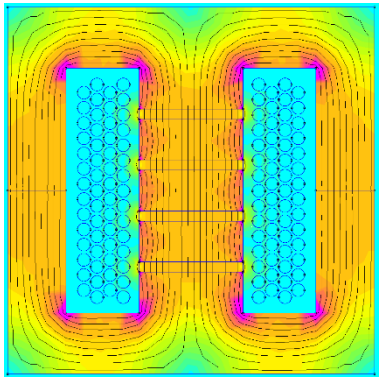
Different air gaps



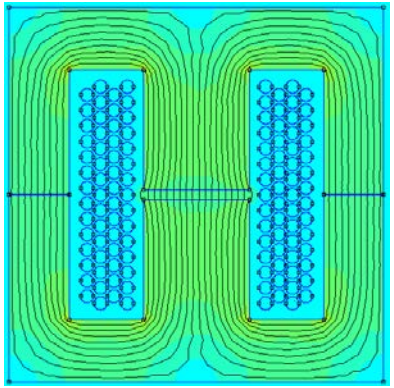
Different distance to air gap



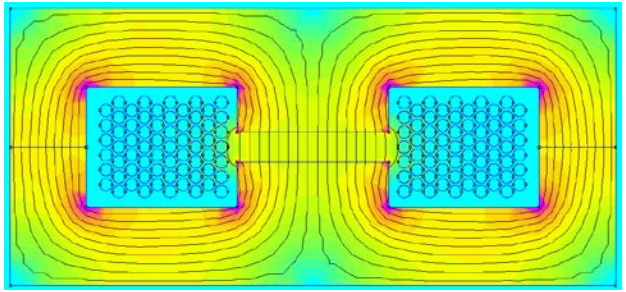
Different frequency



Number of air gaps



Different depth

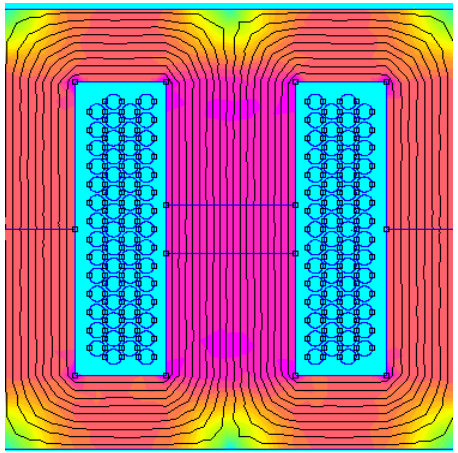


Different window shapes

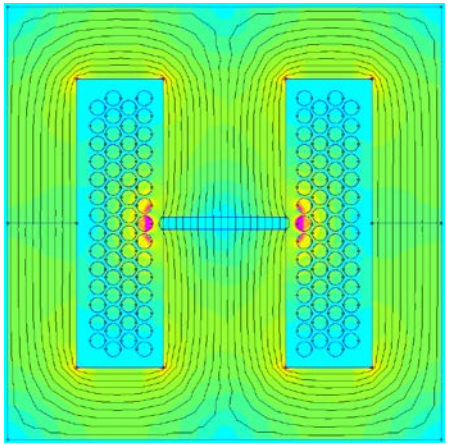
1. http://www.femm.info/wiki/HomePage
2. Martin Grübl, Sumdia Components & Modules GmbH

2D simulation tools for evaluation wire losses at different air gaps [1;2]

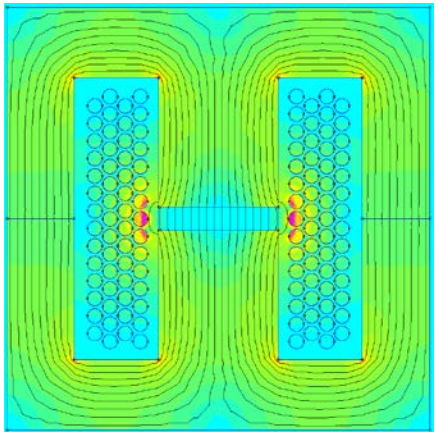
Component:	E36	E36	E36	E36
Wire:	58Wdg, 1,25mm	58Wdg, 1,25mm	58Wdg, 1,25mm	58Wdg, 1,25mm
Air gap:	0 mm	1 mm	2 mm	4 mm
Curent:	0,2 A, 100kHz	0,2 A, 100kHz	0,2 A, 100kHz	0,2 A, 100kHz
L-value:	19,35mH	0,556 mH	0,296 mH	0.157 mH



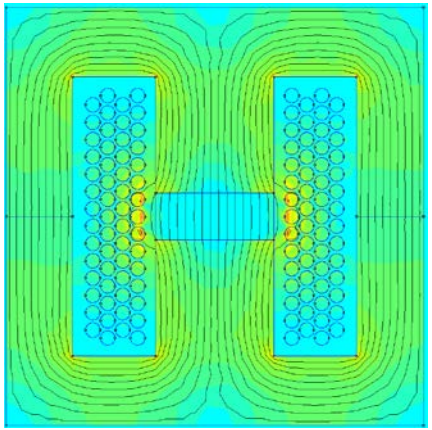
JMAG: 0,0743 W
Model: 0,0623 W
FEMM : 0,0122 W



JMAG: 0,3323 W
Model: 0,3092 W
FEMM : 0,1004 W



JMAG: 0,2996 W
Model : 0,3091 W
FEMM : 0,0936 W



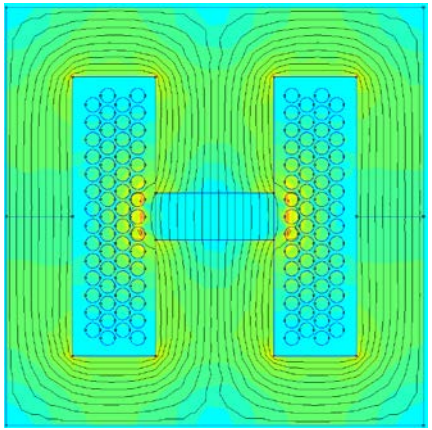
JMAG: 0,2489 W
Model: 0,3169 W
FEMM : 0,0769 W

Gap losses are in first approximation independent from the distance of the air gap (step function)

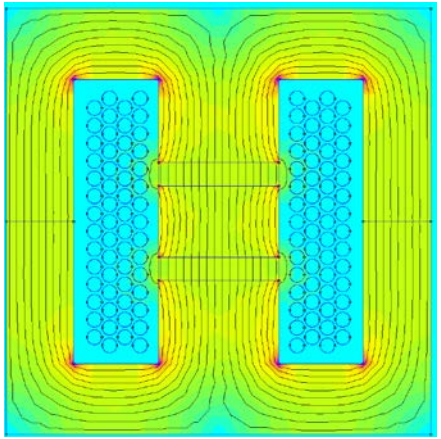
1. http://www.femm.info/wiki/HomePage
2. Martin Grübl, Sumdia Components & Modules GmbH

2D simulation for evaluation wire losses at different number of gaps [1;2]

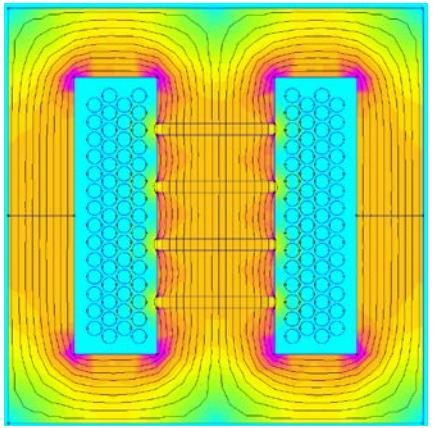
Component:	E36	E36	E36
Wire:	58Wdg, 1,25mm	58Wdg, 1,25mm	58Wdg, 1,25mm
Air gap:	1x4 mm	2x2 mm	4x1 mm
Curent:	0,2 A, 100kHz	0,2 A, 100kHz	0,2 A, 100kHz
L-value:	0.157 mH	0.147 mH	0.139 mH



JMAG: 0,2489 W
Model: 0,3169 W
FEMM : 0,0769 W



JMAG: 0,1241 W
Model: 0,1863 W
FEMM : 0,0389 W



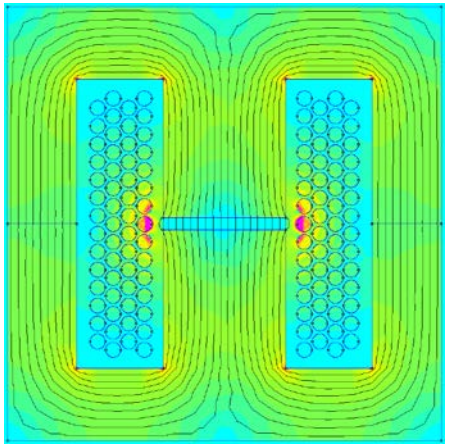
JMAG: 0,0757 W
Model: 0,1168 W
FEMM: 0,0211 W

Gap losses are inversely proportional to number of air gaps: P_{gap}~1/n

1. http://www.femm.info/wiki/HomePage
2. Martin Grübl, Sumdia Components & Modules GmbH

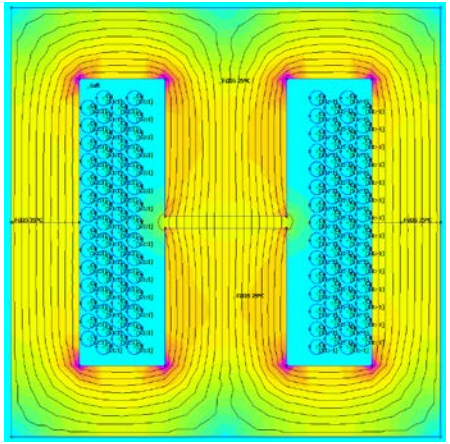
2D simulation tools for evaluation at different distance to gap [1;2]

Component:	E36
Wire:	58Wdg, 1,25mm
Distance to gap:	1mm
Air gap:	1 mm
Current:	0,2 A, 100kHz
L-value:	0,556 mH



JMAG: 0,3323 W
Model: 0,3092 W
FEMM: 0,1004 W

	E36
	58Wdg, 1,25mm
	2mm
	1 mm
	0,2 A, 100kHz
	0,556 mH

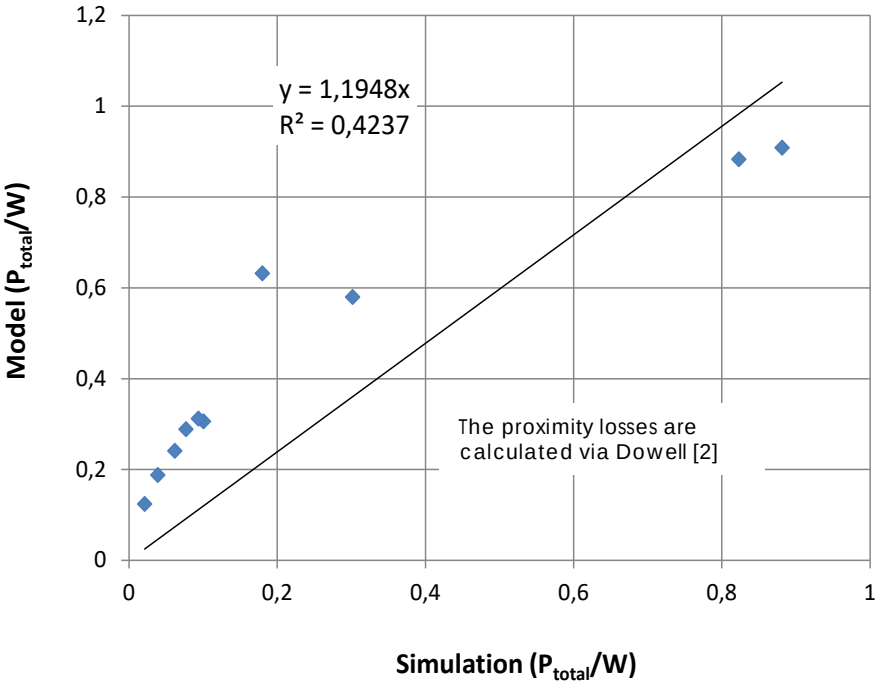


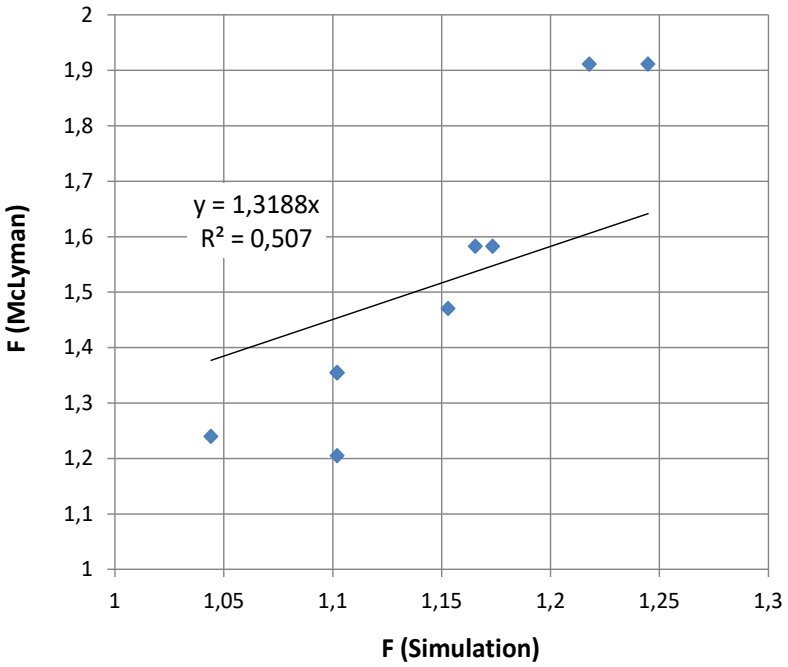
JMAG: 0,2082 W
Model: 0,2211 W
FEMM: 0,0616 W

This increasing distance the losses decrease with: $P_{gap} \sim 1/(1+2n\sigma/s_p)$

1. http://www.femm.info/wiki/HomePage
2. Martin Grübl, Sumdia Components & Modules GmbH

2D simulation result of FEMM [1]

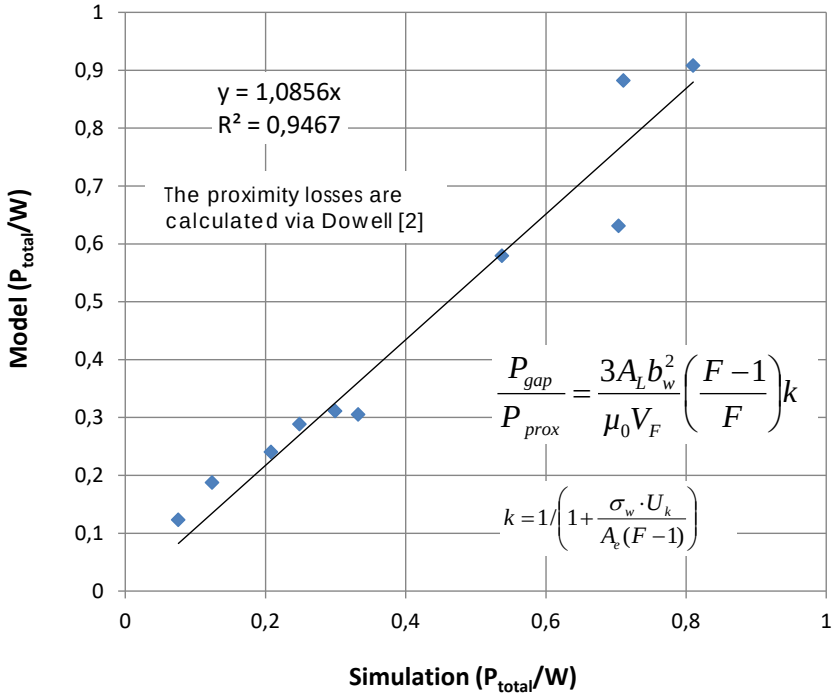


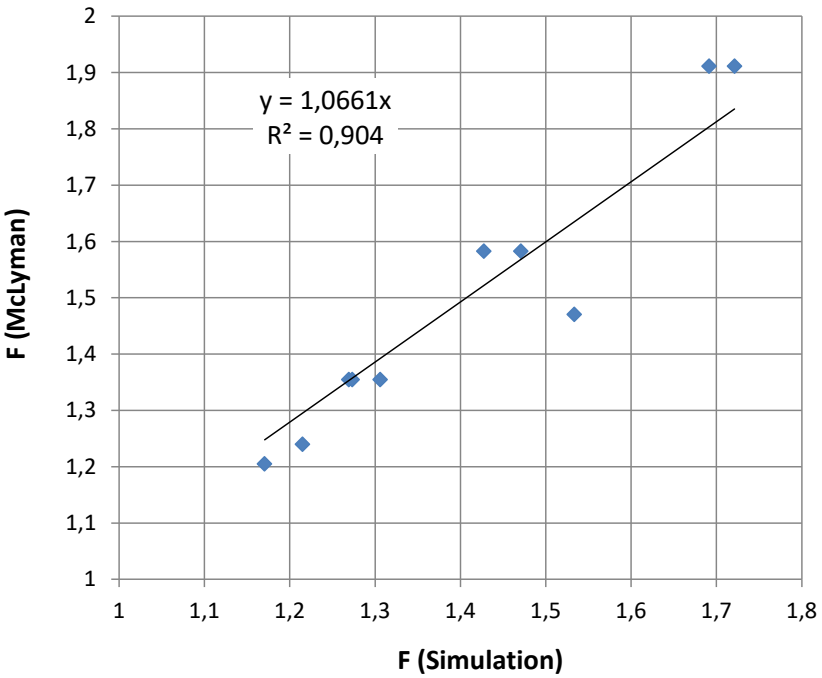


Worse correlation because 2D fringing does not correlate with reality (F-Lyman ≠ F-Simulation)

1. http://www.femm.info/wiki/HomePage
2. Dowell PL (1966) Effects of eddy currents in transformer windings. PROC IEE, Bd 113, Nr 8, S 1387-1394

2D simulation of “JMAG transformer module” with 3D correction [1]





Better correlation because 3D correction improves fringing calculation (F-Lyman ≈ F-Simulation)

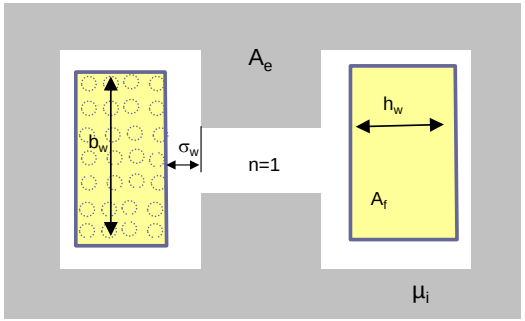
1. Manfred Wohlstreicher, Sumdia Components & Modules GmbH
2. Dowell PL (1966) Effects of eddy currents in transformer windings. PROC IEE, Bd 113, Nr 8, S 1387-1394

Limits and weakness

- The overlay of wire field with the air gap fringing field is neglected
- The different directions of the air gap field are not taken into account
- The model was only tested on round wires

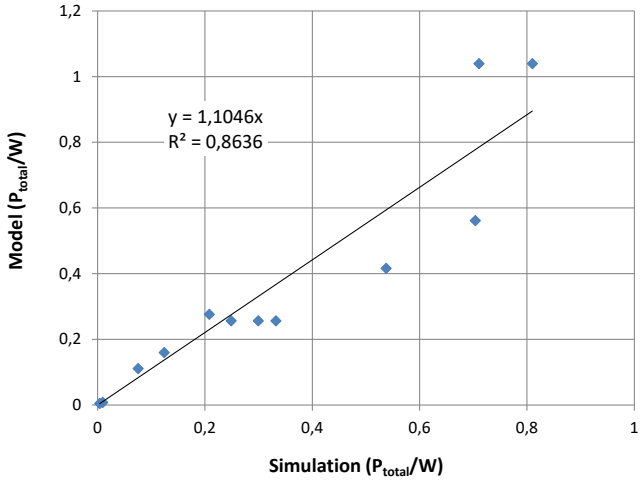
Rule of thumb

$$\frac{P_{gap}}{P_{prox}} \approx \frac{3}{2}\frac{b_w}{h_w n}k$$



$$with: \quad k = \frac{1}{1 + 2n\sigma_w / s_p} \approx 0{,}5$$

P_{gap}: gap losses
P_{gap}: gap losses
b_w: length of winding bulk
h_w: height of winding bulk
n: number of air gaps
s_p: distance of air gap
σ_w: distance to the air gap



Thank you for your attention!